

Matching markets with signals

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Signaling

- ▶ Signaling was introduced in job market for new Ph.D. economists in 2006 year.
- ▶ Each candidate can send signals up to two departments.

What are signals needed for?

- ▶ Reduce congestion (Roth, 2008)
- ▶ Help students with atypical preferences
- ▶ Insure high-ranked students

Congestion

- ▶ **Congestion** is frictions in a decentralized market.
- ▶ Ideal world is a market without frictions.
 - ▶ Departments can make offers following the deferred acceptance algorithm.
- ▶ Real world
 - ▶ time may run out
 - ▶ students may begin accepting offers from other departments
 - ▶ offers may be costly

Related work

- ▶ Coles and Niederle (2007)

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- ▶ Lee and Schwarz (2007a,b)

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- ▶ Lee and Schwarz (2007a,b)
- ▶ Abdulkadiroglu, Che, and Yasuda (2008)

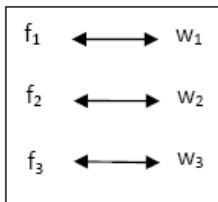
Contribution of this paper

- ▶ *Private* signals facilitate **information asymmetry** that could **harm** *decentralized* matching markets (among firms and workers).
- ▶ The introduction of signals in **almost aligned preference** environment
 - ▶ **decreases** number of matches,
 - ▶ **decreases** total firms' welfare,
 - ▶ **ambiguously** effects total workers' welfare.

compare to no signaling case.

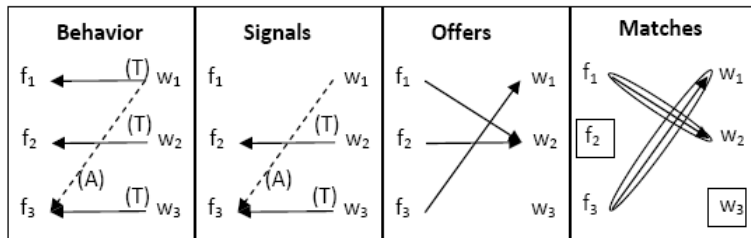
A simple example

- ▶ 3 firms and 3 workers
- ▶ $\theta_{f_j} = (w_1, w_2, w_3)$
- ▶ $\theta_{w_i} = (1 - \varepsilon) \theta_0 \oplus \varepsilon \theta_{a_i}, i.i.d.$
 - ▶ $\varepsilon \ll 1$
 - ▶ $\theta_0 = (f_1, f_2, f_3)$ - "typical"
 - ▶ $\theta_{a_i} \sim U(\Theta_s)$ - "atypical"
- ▶ No signals



A simple example

► With signals



Outline

1. Model.
2. Almost aligned preferences
3. The role of signals: transmit information, facilitate information asymmetry
4. Conclusion.

- ▶ M firms, N workers ($M \leq N$)
- ▶ Ordinal utility
 - ▶ $\theta_f \in \Theta_F$ – firm f 's preference list (strict)
 - ▶ $\theta_w \in \Theta_W$ – worker w 's preference list (strict)
 - ▶ θ_f and θ_w are i.i.d.; $g(\theta)$ is their joint distribution
- ▶ Cardinal utility
 - ▶ Assign *any cardinal utility* $u_a(\cdot, \theta_a) > 0$ consistent with θ_a , $u_a(\emptyset, \theta_a) = 0$

1. Each worker can send *one* signal to some firm. The signals are *private* (observed only by firms who receive them).
2. Each firm makes *one* offer to some worker. A firm can make an offer to any worker.
3. Workers choose offers to accept. The realized matches are observed.

- ▶ Worker w 's strategy is a duple $v_w = (v_w^1, v_w^2)$
 - ▶ $v_w^1 : \Theta_W \rightarrow F$
 - ▶ $v_w^2 : \Theta_W \times 2^F \rightarrow F$
- ▶ Firm f 's strategy $v_f : \Theta_F \times 2^W \rightarrow W$
- ▶ Perfect Bayesian equilibrium

Almost aligned preferences

- ▶ $\forall j, \theta_{f_j} = \hat{\theta} = \text{const}$ ($v_f(h, \theta_f) \equiv v_f(h)$, $h \subset W$)
- ▶ $\forall i, \theta_{w_i} = (1 - \varepsilon) \theta_0 \oplus \varepsilon \theta_{a_i}$, *i.i.d.*
 - ▶ $\theta_0 = \text{const}$ - "typical"
 - ▶ $\theta_{a_i} \sim A(\Theta_s)$ - "atypical"
- ▶ Enumerate firms and workers according to θ_0 and $\hat{\theta}$.

Assumptions.

Assumption. $0 < \varepsilon \ll 1$.

Assumption. $A(\Theta_s)$ has a full support

Assumption (Positive Role of Signals). $\forall f \in F, w \in W,$
 $h \subset W \ v_f(h \setminus w) = w \Rightarrow v_f(h \cup w) = w.$

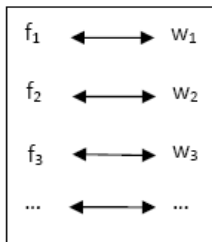
Definition. $h \subset W$ is **reached** for firm f when agents follow $v = (v_w, v_f)$ if $Pr(h_f(\theta) = h | v) > 0$

Definition. Firm f **responds** to worker w 's signal if

- ▶ $\exists h \subset W : h \setminus w$ and $h \cup w$ are *reached* for firm f and
- ▶ $v_f(h \setminus w) \neq v_f(h \cup w)$.

Benchmark. No signals

- ▶ The only equilibrium is the assortative matching.



Signals. The set of equilibria.

Definition.

- ▶ An equilibrium is **babbling** if no firm *responds* to any signal.
- ▶ An equilibrium is **informative** if there is at least one firm that *responds* to some worker's signal.

Babbling equilibria

Theorem

The only possible match in a babbling equilibrium is the assortative match.

Theorem

If each firm responds to all signals from workers better or equal to the corresponding one, the set of strategies

- $v_{w_i}(\theta_{w_i}) = \max_{\theta_{w_i}} (f' \in F : f' \preceq_{\theta_0} f_i)$
- $v_{f_j}(h) = \begin{cases} \max_{\theta_{f_j}} (w : w \in h) & \text{if } h \cap \{w_1, \dots, w_j\} \neq \emptyset \\ w_{j+1} & \text{if } h \cap \{w_1, \dots, w_j\} = \emptyset \end{cases}$

and set of firms' beliefs consistent with agents' strategies constitute a unique equilibrium.

Illustration

Welfare measure.

- ▶ $EMatches(v) = \sum_{\theta} g(\theta) m(v(\theta), \theta)$
 - ▶ $m(v(\theta), \theta)$ is the ex-post number of matches for $v = (v_w, v_f)$ and $\theta \in \Theta_W \times \Theta_F$.
- ▶ $EW_{firms}(v) = \sum_f \sum_{\theta} g(\theta) Pr_f(h_f(\theta)) u_f(v_f(\theta) | v_{-f}(\theta), h_f(\theta), \theta_f)$
- ▶ $EW_{workers}(v) = \sum_w \sum_{\theta} g(\theta) u_w(v_w(\theta_w) | v_{-w}(\theta_{-w}), \theta_w)$

Welfare comparison.

$EMatches$	No signaling \geq Signaling (informative)
$EW_{workers}$	No signaling \leq Signaling (informative)
EW_{firms}	No signaling \geq Signaling (informative)

Welfare comparison.

Theorem

If firm f_{j-1} and at least one firm among $\{f_{j+1}, \dots, f_M\}$ responds to worker w_{j-1} 's signal in an informative equilibrium, then

$EMatches_{f_j}$	No signaling > Signaling (informative)
EW_{f_j}	No signaling > Signaling (informative)

Uniform distribution (Coles and Niederle, 2007) VS almost aligned preferences

Welfare comparison

Preferences	<i>No signals</i>	<i>EMatch</i>	EW_{work}	EW_{firm}
Almost aligned	0	-	±	-
Uniform distr.	0	+	+	±

Uniform distribution (Coles and Niederle, 2007) VS almost aligned preferences

Role of signals

Preferences	Transmit information	Facilitate information asymmetry
Almost aligned	Small	Large
Uniform distr.	Large	Small

- ▶ **Matching** model of the Economics job market **with signals**
 - ▶ there is a "realistic" environment when signals **!!do harm!!** the matching market.
- ▶ The roles of *private* signals in matching markets
 - ▶ transmit information (**positive**)
 - ▶ facilitate information asymmetry (**negative**)

Thank you

