# Lecture Notes on BEJK 2003

Andrés Rodríguez-Clare
PSU and NBER

April 2007

# 1 Introduction

- The simplest way to introduce "firms" is *a la* Krugman.

- In this model each industry variety is produced by a single firm in just one country, which exports its output everywhere else in the world.

- In other words, *all firms export to all countries*.

- This remains true with Iceberg transportation costs.

- This turns out to be inconsistent with the basic facts (from BEJK, as summarized by Antras):
  - Exporters are in the minority. In 1992, only 21% of U.S. plants reported exporting anything.
  - Exporters sell most of their output domestically: around 2/3 of exporters sell less than 10% of their output abroad.
  - Exporters are bigger than non-exporters... even their domestic sales are 4.8 larger than non-exporters.

- Plants are also heterogeneous in measured productivity.
  - Exporters' productivity distribution is a shift to the right of the nonexporter's distribution. Exporters have, on average, a 33% advantage in labor productivity relative to nonexporters.
  - This suggests that the most productive firms self-select into export markets, but it could also reflect learning by exporting (Clerides et al., 1998)

- Micro-level studies have also found evidence of substantial reallocation effects within an industry following trade liberalization episodes.
  - Exposure to trade forces the least productive firms to exit or shut-down (Bernard and Jensen, 1999; Aw, Chung and Roberts, 2000; Clerides et al.,1998).
  - Trade liberalization leads to market share reallocations towards more productive firms, thereby increasing aggregate productivity (Pavcnik, 2002, Bernard, Jensen and Schott 2003).

- Successful theoretical frameworks for studying firms and the decision to export should include two features:

1. Within sectoral heterogeneity in size and productivity

2. A feature that leads only the most productive firms to engage in foreign trade.

- This last feature could be a sunk cost of exporting as documented by Roberts and Tybout (1997) and Bernard and Jensen (2004), and formalized by Melitz (2003), or a limitation on product differentiation (i.e., a fixed measure of goods) that leads to worldwide (price) competition in the production of a particular good, which in turn gives rise to variable markups (BEJK).

# 2 BEJK 2003 (AER)

- This model is compatible with the above results that exports exhibit higher *measured* productivity, and that exporters have larger domestic sales.

- The model is basically a static version of EK 2001 (i.e., firms own technologies and engage in Bertrand competition) extended to CES preferences.

- If the elasticity of substitution $\sigma \leq 1$ then there is no significant change, whereas if $\sigma > 1$ then the mark-up is caped above at $\overline{m} = \frac{1}{1-1/\sigma}$. This just introduces a minor complication in some formulas.

## 2.1 The Joint Frechet in BEJK

In BEJK they assume that the best and second best idea in each country come from a joint Frechet distribution.

Formally, letting $Z_{ki}$ be the kth best idea in country $i$, then we have (for $0 \leq z_2 \leq z_1$)

$$F_i(z_1, z_2) = \Pr[Z_{1i} \leq z_1, Z_{2i} \leq z_2] = \left[1 + T_i \left(z_2^{-\theta} - z_1^{-\theta}\right)\right] e^{-T_i z_2^{-\theta}}$$

It turns out that this is exactly the distribution of the first and second best idea in the case where ideas come at a stochastic process like in EK 2001, with a Poisson rate of arrival of ideas at time $t$ of $\mu_i L_{it}$ (in EK 2001 $\mu_i = \alpha_i r_i$), and with the quality of ideas distributed Pareto with parameter $\theta$, $H(z) = 1 - z^{-\theta}$, $z \geq 1$.

To see this, drop subindices, and let $T = \mu \int_0^t L_s ds$ be the stock ideas.

We know that the quantity of ideas for any good is distributed Poisson with mean $T$.

Imagine first that there are $k$ ideas, ordered as idea 1, idea 2, up until idea $k$. Let $Z(j)$ be the quality of idea $j$ (note: here I am using $j$ to distinguish among ideas, not among goods... this is just for this argument, then I revert below to the BEJK notation).

What is the probability that the quality of the best idea is lower than or equal to $z_1$, whereas the second best idea is lower than $z_2 \leq z_1$?

We have $k + 1$ possibilities: the first $k$ correspond to the case that $Z(j) \in [z_2, z_1]$ for some $j$ and $Z(l) \leq z_2$ for all other $l \neq j$, and the $k + 1$ possibility corresponds to the case that $Z(j) \leq z_2$ for all $j$.

The first $k$ possibilities happen with probability

$$k \left[H(z_1) - H(z_2)\right] H(z_2)^{k-1}$$

whereas the last possibility has probability $H(z_2)^k$.

Now we take into consideration all possible $k$, including $k = 0$, for which the only relevant probability is $H(z_2)^k = 1$. Thus, we have

$$\begin{aligned}
\Pr[Z_1 &\leq z_1, Z_2 \leq z_2] \\
&= \sum_{k=1}^{\infty} \left( \frac{e^{-T} T^k}{k!} \right) k \left[ H(z_1) - H(z_2) \right] H(z_2)^{k-1} + \sum_{k=0}^{\infty} \left( \frac{e^{-T} T^k}{k!} \right) H(z_2)^k \\
&= e^{-T} T \left[ H(z_1) - H(z_2) \right] \sum_{k=1}^{\infty} \frac{(T H(z_2))^{k-1}}{(k-1)!} + e^{-T} \sum_{k=0}^{\infty} \frac{(T H(z_2))^k}{k!} \\
&= e^{-T} \left( T \left[ H(z_1) - H(z_2) \right] e^{T H(z_2)} + e^{T H(z_2)} \right) \\
&= \left( 1 + T(z_2^{-\theta} - z_1^{-\theta}) \right) ) e^{-T z_2^{-\theta}} = F(z_1, z_2, T)
\end{aligned}$$

## 2.2 The Joint Cost Distribution in BEJK

Let $C_{kni}$ be the k-th least cost of delivering the good from $i$ to $n$, and is given by $C_{kni} = w_i d_{ni}/Z_{ki}$.

The least cost of delivering a good to market $n$, $C_{1n}$ is given by $\min_i\{C_{1ni}\}$.

Letting $i^* = \arg\min\{C_{1ni}\}$, then $C_{1n} = C_{1ni^*}$.

The second least cost of delivering a good to market $n$ is either associated with the country with the runner up among best ideas, $\min_{i \neq i^*}\{C_{1ni}\}$ ,or with the second best idea in country $i^*$, $C_{2ni^*}$. Thus, finally we see that

$$C_{2n} = \min\left\{C_{2ni^*}, \min_{i \neq i^*}\{C_{1ni}\}\right\}$$

We want to obtain the joint cost distribution in equation (8), namely

$$\begin{aligned} G_n(c_1, c_2) &= \Pr[C_{1n} \leq c_1, C_{2n} \leq c_2] \\ &= 1 - e^{-\Phi_n c_1^\theta} - \Phi_n c_1^\theta e^{-\Phi_n c_2^\theta} \quad \text{for} \quad c_1 \leq c_2 \end{aligned}$$

where $\Phi_n = \sum T_i (w_i d_{ni})^{-\theta}$.

We start with $G_{ni}(c_1, c_2)$ - the joint cost distribution if the good is procured from country $i$. In fact, it is convenient to work with the complement of this distribution, namely

$$G_{ni}^c(c_1, c_2) = \Pr[C_{1ni} \geq c_1, C_{2ni} \geq c_2]$$

Since $C_{kni} = w_i d_{ni} / Z_{ki}$, then

$$
\begin{aligned}
\Pr\left[C_{1ni} \geq c_1, C_{2ni} \geq c_2\right] &= \Pr\left[w_i d_{ni}/Z_{1i} \geq c_1, w_i d_{ni}/Z_{2i} \geq c_2\right] \\
&= \Pr\left[Z_{1i} \leq w_i d_{ni}/c_1, Z_{2i} \leq w_i d_{ni}/c_2\right] \\
&= F(w_i d_{ni}/c_1, w_i d_{ni}/c_2, T_i) \\
&= \left(1 + T\left(w_i d_{ni}\right)^{-\theta}\left(c_2^{\theta} - c_1^{\theta}\right)\right) e^{-T\left(w_i d_{ni}\right)^{-\theta} c_2^{-\theta}} \\
&= F(c_1^{-1}, c_2^{-1}, T\left(w_i d_{ni}\right)^{-\theta})
\end{aligned}
$$

Now consider the complementary distribution for the lowest and second-lowest cost of delivering a good to market $n$, without regard to its source, $G_n^c(c_1, c_2) = \Pr[C_{1n} \geq c_1, C_{2n} \geq c_2]$.

One way in which this is satisfied is if the lowest and second-lowest cost from all countries are higher than $c_2$, which has probability $\Pi_i G_{ni}^c(c_2, c_2)$.

Then there is the possibility that for country $i$ we have $C_{1ni} \in [c_1, c_2[$ and $C_{2ni} \geq c_2$, together with $C_{1nk} \geq c_2$ for all $k \neq i$.

Thus,

$$\begin{aligned}
G_n^c(c_1, c_2) &= \prod_i G_{ni}^c(c_2, c_2) + \sum_i \left[G_{ni}^c(c_1, c_2) - G_{ni}^c(c_2, c_2)\right] \prod_{k \neq i} G_{nk}^c(c_2, c_2) \\
&= \prod_i e^{-T_i (w_i d_{ni})^{-\theta} c_2^\theta} \\
&\quad + \sum_i T_i (w_i d_{ni})^{-\theta} (c_2^\theta - c_1^\theta) e^{-T_i (w_i d_{ni})^{-\theta} c_2^\theta} \prod_{k \neq i} e^{-T_k (w_k d_{nk})^{-\theta} c_2^\theta} \\
&= \left[1 + \phi_n (c_2^\theta - c_1^\theta)\right] e^{-\phi_n c_2^\theta} \\
&= F(c_1^{-1}, c_2^{-1}, \phi_n)
\end{aligned}$$

To get $G_n(c_1, c_2)$ note that there are two alternative (non-intersecting) ways for $C_{1n} \leq c_1, C_{2n} \leq c_2$ to be violated:

1) that $C_{1n} > c_1$ irrespective of what happens with $C_{2n}$, which has probability $G_n^c(c_1, c_1)$, and

2) that $C_{1n} \leq c_1$ but $C_{2n} > c_2$, which has probability $G_n^c(0, c_2) - G_n^c(c_1, c_2)$.

Thus,

$$G_n(c_1, c_2) = 1 - G_n^c(c_1, c_1) - G_n^c(0, c_2) + G_n^c(c_1, c_1)$$

Plugging in from above we get the cost distribution in (8).

## 2.3 Efficiency and measured productivity

Even though productivity is $z$, measured labor productivity is something different, namely $pq/L(q)$, where $p$ is the price charged, $q$ is the total quantity produced and sold, and $L(q)$ is the total quantity of labor used.

If true productivity is $z$, we know that $q = zL(q)$. Thus,

$$\frac{pq}{L(q)} = \frac{pz}{w}w = \frac{p}{w/z}w = mw$$

where I have used that the mark-up is $m = p/c = p/(w/z)$.

Thus, if the world behaves according to this model then measured productivity differs across firms only because of differences in the mark-up $m$.

BEJK show that measured productivity is higher in firms with a higher true productivity. This entails showing that the mark-up $m$ charged in the domestic market is increasing in a firm's true productivity $z$.

To do this, note that the distribution of the mark-up in country $n$, $M_n = C_{2n}/C_{n1}$, conditional on $C_{1n} = c_1 \geq 0$, is given by (for $m \geq 1$ and having $g_n(c_1, c_2)$ denote the joint density corresponding to $G_n$)

$$\begin{aligned}
\Pr[M_n \leq m \mid C_{1n} = c_1] &= \Pr[C_{2n}/c_1 \leq m \mid C_{1n} = c_1] \\
&= \frac{\Pr[C_{2n} \leq mc_1 \wedge C_{1n} = c_1]}{\Pr[C_{1n} = c_1]} \\
&= \frac{\int_{c_1}^{mc_1} g_n(c_1, c_2) dc_2}{\int_{c_1}^{\infty} g_n(c_1, c_2) dc_2} \\
&= 1 - e^{-\Phi_n c_1^{\theta}(m^{\theta} - 1)}
\end{aligned}$$

Suppose that good $j$ is supplied by a producer from country $n$.

If the efficiency of this producer is higher then $c_1$ is lower, which from the above result implies that the probability of a low mark-up decreases.

Thus, goods domestically supplied by more efficient firms are associated with higher mark-ups.

## 2.4 Efficiency and exporting

BEJK show that exporters are likely to be more efficient than non-exporters. This is easy to show.

A firm in country $i$ can sell in its domestic market even if its unit cost is higher than the unit cost of a firm in country $k$ because of the "protection" afforded to it by transportation cost $d_{ik}$.

In fact, a firm in country $i$ can survive competition in its domestic market from a firm in country $k$ even if its cost is $d_{ik}$ higher.

But consider this same competition in a third market $n$.

There the "protection" afforded by differential transportation costs is $d_{nk}/d_{ni}$.

But from the triangular inequality we know that $d_{nk} \leq d_{ni}d_{ik}$, hence $d_{nk}/d_{ni} \leq d_{ik}$.

This implies the reasonable result that protection afforded by transportation costs for sales in a firm's domestic market is higher than in any other market.

Thus, firms that export must be more efficient than non-exporters.

## 2.5 Efficiency and domestic sales

Finally, BEJK show that exporters on average sell more in their domestic market.

Note that this is not just saying that exporters have larger sales, which would not be surprising since they sell in more markets. What they are saying is that exporters have larger domestic sales than non-exporters.

This result requires assuming that $\sigma > 1$. Under this assumption, what is required is to show that exporters charge a lower price (on average) in their domestic market than non-exporters.

Note that although we know that exporters have a lower cost, we also know that they charge higher mark-ups, so it is not immediately clear what happens to the price they charge.

An easy case is the one in which $m = \overline{m}$, since in that case mark-ups are not higher with $z$ and hence a higher $z$ does imply a lower cost and a lower price.

But imagine that $m < \overline{m}$. Since the mark-up charged in market $n$ is $m_n = C_{2n}/C_{1n}$, and the firm selling there has cost $C_{1n}$, then the price it charges is $C_{2n}$.

Hence, we need to show that exporters face (on average) rivals with lower costs (i.e., lower $C_{2n}$).

But note that the conditional distribution of $C_{2n}$ given $C_{1n} = c_1$ is

$$\begin{aligned}
\Pr\left[C_{2n} \leq c_2 \mid C_{1n} = c_1\right] &= \frac{\Pr\left[C_{2n} \leq c_2 \wedge C_{1n} = c_1\right]}{\Pr\left[C_{1n} = c_1\right]} \\
&= \frac{\int_{c_1}^{c_2} g_n(c_1, c_2) dc_2}{\int_{c_1}^{\infty} g_n(c_1, c_2) dc_2} \\
&= 1 - e^{-\Phi_n (c_2^\theta - c_1^\theta)}
\end{aligned}$$

A higher $z$ implies a lower $c_1$ and hence a higher probability of a low $C_{2n}$.

# 3 Summing up

This model is consistent with not all firms exporting, and with exporters having a higher measured productivity and selling more in the domestic market than non-exporters.

The key elements are heterogeneity in efficiency (differences in $z$), endogenous mark-ups (so that firms with a higher $z$ can have higher mark-ups), transportation costs (so that firms with low $z$ can survive in the domestic market).

Note also that there is no differentiation, so nothing needs to "be done" to prevent low productivity firms from selling abroad.