# Lecture Notes for Eaton and Kortum (2001)

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Start at Section 3.

Note: there is a typo in Section 2, equation (5), instead of $P_n = \gamma \phi_n^{-1/\theta}$ it should be $P_n = e^{-\gamma/\theta} \phi_n^{-1/\theta}$.

Main difference with what we have seen before is that ideas are the result of purposeful research, done by innovators in the expectation of future profits. Thus, ideas are now privately owned.

Imagine that an entrepreneur from country $i$ has an idea with quality $q$. The cost of selling in market $n$ is $w_i d_{ni}/q$.

The probability that he will be able to charge a mark-up of at least $m$ is then equal to the probability that the least cost for that good in country $n$ is no lower than $m w_i d_{ni}/q$.

But recall that the least cost in country $n$ is distributed $\Pr[c_n \leq c] = G_n(c) = 1 - \exp[-\phi_n c^\theta]$.

Hence, the probability that an entrepreneur with an idea of quality $q$ will be able to charge a mark-up of at least $m$ in country $n$ is $1 - G_n(m w_i d_{ni}/q)$.

Recall that $q$ is distributed Pareto with parameter $\theta$, i.e. $H(q) = \Pr[Q \leq q] = 1 - q^{-\theta}$ for $q \geq 1$.

Hence, the probability that an entrepreneur in $i$ with an idea of unknown quality drawn from this distribution will be able to charge a mark-up of at least $m$ in country $n$ is (using $a \equiv w_i d_{ni}$)

$$
\begin{aligned}
b_{ni}(m) &= \int_1^{\infty} \left[1 - G_n(am/q)\right] dH(q) \\
&= \int_1^{\infty} e^{-\phi_n (am)^{\theta} q^{-\theta}} \theta q^{-\theta-1} dq
\end{aligned}
$$

Using $x = am/q$ then $dq = -amx^{-2}dx$, and hence

$$\begin{aligned} b_{ni}(m) &= \int_1^{\infty} e^{-\phi_n x^{\theta}} \theta \left(am/x\right)^{-\theta-1} amx^{-2} dx \\ &= \frac{1}{\phi_n \left(am\right)^{\theta}} \int_0^{\infty} e^{-\phi_n x^{\theta}} \phi_n \theta x^{\theta-1} dx \\ &\quad - \int_0^1 e^{-\phi_n x^{\theta}} \theta (am)^{-\theta} x^{\theta-1} dx \end{aligned}$$

The integral in the first term is one, since it is just the integral of the density of the distribution $G_n(c)$. The second term becomes arbitrarily small as $\phi_n$ is large, which happens as $T$ becomes sufficiently high. So Kortum (1997) and Eaton and Kortum (2001) use the approximation

$$b_{ni}(m) \approx \frac{1}{\phi_n \left(mw_i d_{ni}\right)^{\theta}}$$

Given the definition of $b_{ni}(m)$ then $b_{ni}(1)$ is the probability that an idea of unknown quality in country $i$ will "have a market" in country $n$, i.e. the probability that it will survive the competition, which entails charging a mark-up of at least one.

Recalling the expression $\pi_{ni} = T_i(w_i d_{ni})^{-\theta}/\phi_n$, then this probability can be written as

$$b_{ni}(1) = \frac{\pi_{ni}}{T_i}$$

For an idea to make it in country $n$, it must pass two hurdles: first, it has to be the best idea in country $i$, which happens with probability $1/T_i$, and second, it has to beat the competition from the rest of the world, which means being the least cost way of supplying country $n$, which has probability $\pi_{ni}$.

We have seen that the probability that an idea of unknown quality in $i$ entails a mark-up of at least $m$ in country $n$ is $b_{ni}(m)$.

The probability that such an idea has a market but with a mark up no higher than $m$ is $b_{ni}(1) - b_{ni}(m)$, and the conditional probability that the mark-up is no higher than $m$ given that it has a market in country $n$ is

$$\frac{b_{ni}(1) - b_{ni}(m)}{b_{ni}(1)} = 1 - m^{-\theta} = H(m)$$

This says that the mark-up among active ideas is distributed Pareto with parameter $\theta$. This does not depend on the country of origin, or destination, and it doesn't change in time. This implies that $\pi_{ni}$ continues to be the share of spending in country $n$ devoted to goods coming from country $i$. Hence, the trade balance condition remains

$$\sum_n \pi_{ni} Y_n = Y_i$$

EK 2001 have Cobb-Douglas preferences, with equal shares across goods, so if a firm sells a good with at price $p$ and cost $c$ in a country with income $Y$, it makes profits of $Y - (Y/p)\,c = Y(1 - m^{-1})$.

The expected profits of an idea from country $i$ that beats the competition in country $n$ are then

$$
\begin{aligned}
Y_n \int_1^{\infty} (1 - m^{-1}) dH(m) &= Y_n \int_1^{\infty} dH(m) - Y_n \int_1^{\infty} m^{-1} \theta m^{-\theta-1} dm \\
&= Y_n \left(1 - \frac{\theta}{\theta+1}\right) = \beta Y_n
\end{aligned}
$$

where $\beta \equiv 1/(1+\theta)$.

The share of goods that country $n$ buys from country $i$ is $\pi_{ni}$. Hence, total profits earned by entrepreneurs in country $i$ are

$$\sum_n \beta \pi_{ni} Y_n = \beta Y_i$$

where the second term follows because of the trade-balance condition.

This implies that $\beta$ is the profit share in every country.

Thanks to this property, the static analysis done in EK 2002, with perfect competition and no profits, remains valid for the period by period equilibrium in the dynamic model in EK 2001 with Bertrand competition and quasi-profits.

To see this in more detail, let's show that the trade balance condition stated in terms of wages, i.e. $\sum_n \pi_{in} w_n L_n^p = w_i L_i^p$, where $L_i^p$ is number of workers engaged in production (as opposed to research), continues to hold.

Note that $Y_i = w_i L_i = w_i L_i^p + \beta Y_i$ implies $Y_i = w_i L_i^p / (1 - \beta)$.

This implies that $\sum_n \pi_{ni} Y_n = Y_i$ is equivalent to $\sum_n \pi_{ni} w_n L_n^p = w_i L_i^p$.

Let's now consider time explicitly. The intertemporal utility function is

$$u_t = \int_0^\infty e^{-\rho(s-t)} U_s ds$$

These preferences pin down the real interest rate at $\rho$, so it is appropriate to discounte future profits by the discount rate.

The real expected discounted profits made in country $n$ of an idea from country $i$ at time $t$ is given by (using $b_{nis} = b_{nis}(1)$)

$$V_{nit}/P_{it} = \int_t^\infty e^{-\rho(s-t)} (\beta b_{nis} Y_{ns}/P_{is}) ds$$

We then have

$$V_{it} = \sum_n V_{nit} = \beta \int_t^\infty e^{-\rho(s-t)} \left(P_{it}/P_{is}\right) \left(1/T_{is}\right) \sum_n \pi_{nis} Y_{ns} ds$$

But $\sum_n \pi_{nis} Y_{ns} = Y_{is}$, whereas $P_{it}$ falls at a rate equal to $\theta g_L$, hence $P_s = P_t e^{-(g_L/\theta)(s-t)}$ and

$$V_{it} = \beta \int_t^\infty e^{-(\rho - g_L/\theta)(s-t)} (Y_{is}/T_{is}) ds$$

Using $Y_{it} = w_{it}(1 - r_i)L_{it}/(1 - \beta)$ and $T_{it} = (\alpha_i r_i / g_L)L_{it}$ then we get

$$V_{it} = \frac{\beta}{1-\beta} \int_t^{\infty} e^{-(\rho - g_L/\theta)(s-t)} \left( \frac{w_{is}(1 - r_i)L_{is}}{(\alpha_i r_i / g_L)L_{is}} \right) ds$$

Assuming that $\rho > g_L/\theta$, and noting that in steady state wages are constant, then

$$\begin{aligned} V_i &= (1/\theta) \left( \frac{w_i(1 - r_i)}{(\alpha_i r_i / g_L)} \right) \left( \frac{1}{\rho - g_L/\theta} \right) \\ &= \left( \frac{g_L}{\alpha_i r_i} \right) \frac{w_i(1 - r_i)}{\theta\rho - g_L} \end{aligned}$$

In equilibrium the expected payoff to research must be equal to the wage in every country. This entails, $\phi_i V_i = w_i$. This can be solved to yield

$$r_i = g_L/\theta\rho \text{ for all } i$$

This implies that differences in $\alpha_i$ do not affect the proportion of workers engaged in research.

On the other hand, a lower $\rho$ and a lower $\theta$ lead to a higher proportio nof people doing research.

Moreover, research intensity does not depend on size or openness... why?

Finally, recall that wages depend on $T_i/L_i = \alpha_i r_i/g_L = \alpha_i/\theta\rho$. So research productivity does affect wages through its *direct* effect on $T/L$.