Lecture Notes for Eaton, Kortum and Kramarz (2005)

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- Stylized facts regarding firms and exports: few firms export, exporters sell most of their output domestically, exporters have higher measured productivity, exporters have larger domestic sales.

- Two models have been proposed: Ricardian models (BEJK) with Iceberg transportation costs and Monopolistic Competition (Melitz) with fixed costs of exporting.

- It is important to know which model is better, or whether we need elements of both. For this, we need a combined model.

- Data on French firms (manufacturing, 1986) exports across destinations and in the domestic market.

  - Consistent with BEJK, firms that sell in more markets sell more in France.

  - But more French firms sell in larger markets... variety of imports increases with market size. We need variety to be endogenous, adding fixed costs.

The point of departure is that now the measure of goods is J, so $z_i(j)$ comes from

$$F_i(z) = \Pr\left[Z_i \leq z\right] = \exp\left[-(T_i/J)z^{-\theta}\right]$$

and the measure of goods that can be produced in country i with $Z \geq z$ is

$$J\left\{1 - \exp\left[-(T_i/J)z^{-\theta}\right]\right\}$$

Later they show that if $J = 1$ and no fixed costs then this collapses to EK 2002 whereas if $J \to \infty$ then this collapses to monopolistic competition like Melitz or Chaney.

The distribution of costs is

$$\Pr[C_n \leq c] = 1 - \exp\left[-(\phi_n/J)c^{\theta}\right]$$

and the measure of goods that are potentially supplied to a country n at a cost less than c is

$$\mu_n(c) = J\left\{1 - \exp\left[-(\phi_n/J)c^{\theta}\right]\right\}$$

Fixed costs for good j in market n are *independent of source* and given by

$$E_n(j) = E_n \varepsilon_n(j)$$

Preferences are CES with $1 < \sigma < 1 + \theta$ so

$$X_n(j) = \alpha_n(j) \left(p_n(j)/P_n\right)^{1-\sigma} X_n$$

and

$$P_n^{1-\sigma} = \int_0^J \alpha_n(j) p_n(j)^{1-\sigma} dj$$

Note that just as $\varepsilon_n(j)$, $\alpha_n(j)$ is a shock that is specific to a good in a destination market, but independent of source.

Because of the fixed cost, only the firm with the lowest cost in country n will enter (if it does), so it can behave as a monopolist, charging mark-up $\overline{m} = \sigma/(\sigma - 1)$. The unit price is then $p_n(j) = \overline{m} c_n(j)$ if good j is sold in country n.

Entry will take place as long as profits $(X_n(j)/\sigma)$ are larger than the fixed cost. This entails

$$\alpha_n(j)\left(\overline{m}c_n(j)/P_n\right)^{1-\sigma} X_n \geq \sigma E_n \varepsilon_n(j)$$

Using $\eta_n(j) = \alpha_n(j)/\varepsilon_n(j)$ and $x_n = X_n/\sigma E_n$ this is

$$\eta_n(j)x_n \geq \left(\overline{m}c_n(j)/P_n\right)^{\sigma-1}$$

Given η, entry happens if

$$c_n(j) \leq \bar{c}_n(\eta) \equiv \eta^{1/(\sigma-1)}\bar{c}_n$$

where

$$\bar{c}_n = x_n^{1/(\sigma-1)} P_n/\overline{m} \qquad (1)$$

Integrating across the range of costs in any location n , and assuming first that α and η are fixed, then the price index is

$$P_n^{1-\sigma} = \overline{m}^{1-\sigma} \left\{ \int_0^{\bar{c}_n(\eta)} \alpha c^{1-\sigma} d\mu_n(c) \right\}$$

Imagine now that α is a random variable with density $f(\alpha)$, independent of c, then $P_n^{1-\sigma}$ is equal to

$$\overline{m}^{1-\sigma} \left\{ \int \left(\int_0^{\bar{c}_n(\eta)} \alpha c^{1-\sigma} d\mu_n(c) \right) df(\alpha) \right\} = \overline{m}^{1-\sigma} \left\{ E[\alpha] \left(\int_0^{\bar{c}_n(\eta)} c^{1-\sigma} d\mu_n(c) \right) \right\}$$

Allowing for η to be random as well, then since α and η are not independent, then

$$P_n^{1-\sigma} = \overline{m}^{1-\sigma} \left\{ E_\eta \left[E[\alpha \mid \eta] \int_0^{\eta^{1/(\sigma-1)}\bar{c}_n} c^{\theta-\sigma} \exp\left[-(\phi_n/J)c^\theta \right] \theta\phi_n d(c) \right] \right\} \tag{2}$$

We have two equations in two unknowns: $\bar{c}_n$ and P_n in equations (1) and (2).

The first equation has $\bar{c}_n$ increasing in P_n because a higher P_n implies higher sales for any given cost, hence the maximum cost with positive profits is higher.

The second equation has P_n decreasing in $\bar{c}_n$ because a higher cost implies more firms will make positive profits, hence variety will be higher.

In equilibrium P_n is affected by two things: variety and costs. Costs are captured by ϕ_n. To capture variety, EKK define a new variable, $\widetilde{P}_n$, as

$$\widetilde{P}_n = (P_n/\overline{m})^{1-\sigma}\phi_n^{1-\widetilde{\theta}}$$

where $\widetilde{\theta} \equiv \theta/(\sigma - 1)$

Combining equations (1) and (2) yields (to simplify, in this derivation I assume that $\eta = \alpha = 1$, but the extension is straightforward)

$$(P_n/\overline{m})^{1-\sigma} = \int_0^{(\eta x_n)^{1/(\sigma-1)} P_n/\overline{m}} c^{\theta-\sigma} \exp\left[-(\phi_n/J)c^{\theta}\right] \theta\phi_n dc$$

Using $s = \phi_n c^{\theta}$ implies $ds = \phi_n \theta c^{\theta-1} dc$ so $\theta\phi_n dc = c^{1-\theta} ds$, so

$$(P_n/\overline{m})^{1-\sigma} = \int_0^{\phi_n \left[(\eta x_n)^{1/(\sigma-1)} P_n/\overline{m}\right]^{\theta}} c^{\theta-\sigma} \exp\left[-s/J\right] c^{1-\theta} ds$$

Using $\widetilde{P}_n = (P_n/\overline{m})^{1-\sigma}\, \phi_n^{-1/\widetilde{\theta}}$ we get $(P_n/\overline{m})^{\theta} = \left(\phi_n^{1/\widetilde{\theta}} \widetilde{P}_n\right)^{\theta/(1-\sigma)}$ and hence

$$
\begin{aligned}
\phi_n \left[(\eta x_n)^{1/(\sigma-1)}\, P_n/\overline{m}\right]^{\theta} &= \phi_n\, (\eta x_n)^{\widetilde{\theta}}\, (P_n/\overline{m})^{\theta} \\
&= \phi_n\, (\eta x_n)^{\widetilde{\theta}} \left(\phi_n^{1/\widetilde{\theta}} \widetilde{P}_n\right)^{-\widetilde{\theta}} \\
&= \left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}}
\end{aligned}
$$

so

$$(P_n/\overline{m})^{1-\sigma} = \int_0^{\left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}}} c^{1-\sigma} \exp\left[-s/J\right] ds$$

But $c = (s/\phi_n)^{(1-\sigma)/\theta} = (s/\phi_n)^{-1/\widetilde{\theta}}$, hence

$$(P_n/\overline{m})^{1-\sigma} = \phi_n^{1/\widetilde{\theta}} \int_0^{\left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}}} s^{-1/\widetilde{\theta}} \exp\left[-s/J\right] ds$$

so using again $\widetilde{P}_n = (P_n/\overline{m})^{1-\sigma}\, \phi_n^{-1/\widetilde{\theta}}$ we finally get

$$\widetilde{P}_n = \int_0^{\left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}}} s^{-1/\widetilde{\theta}} \exp\left[-s/J\right] ds$$

Now use $t = s/J$ and get

$$\begin{aligned} \widetilde{P}_n &= \int_0^{\left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}}/J} (tJ)^{-1/\widetilde{\theta}} \exp\left[-t\right] J dt \\ &= J^{1-1/\widetilde{\theta}} \int_0^{\left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}}/J} t^{-1/\widetilde{\theta}} \exp\left[-t\right] dt \\ &= J^{1-1/\widetilde{\theta}} \Gamma\left(1 - 1/\widetilde{\theta}, \left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}}/J\right) \end{aligned}$$

where $\Gamma(a, x) = \int_0^x t^{a-1} e^{-t} dt$ is the incomplete gamma function. This is equation EKK-9 when $\alpha = \varepsilon = 1$.

This defines a function $\widetilde{P}_n(x_n)$, which then allows us to write P_n as

$$P_n = \overline{m} \left[\widetilde{P}_n(x_n) \right]^{-1/(\sigma-1)} \phi_n^{-1/\theta} \tag{3}$$

Note that the relationship between P_n and ϕ_n is the same as in EK 2002. So gravity is still as in EK 2002.

More importantly, the gains from trade are exactly as in EK 2002.

The measure of entrants given η is

$$J\left\{1-\exp\left[-(\phi_n/J)\eta^{\widetilde{\theta}}\bar{c}_n^{\theta}\right]\right\}$$

But using equation (3) then $\bar{c}_n = x_n^{1/(\sigma-1)} P_n/\bar{m}$ implies

$$\bar{c}_n^{\theta} = x_n^{\widetilde{\theta}} \widetilde{P}_n^{-\widetilde{\theta}} \phi_n^{-1}$$

and hence the measure of entrants is

$$\begin{aligned} & J\left\{1-E_\eta \exp\left[-(\phi_n/J)\eta^{\widetilde{\theta}} x_n^{\widetilde{\theta}} \widetilde{P}_n^{-\widetilde{\theta}} \phi_n^{-1}\right]\right\} \qquad (4) \\ = \; & J\left\{1-E_\eta \exp\left[-\left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}}/J\right]\right\} \end{aligned}$$

Conditional on entry, suppliers from all countries have the same cost distribution in market n, so (given common mark-up) they have the same distribution of prices.

The share of X_n allocated to country i is then π_{ni}.

Moreover, since on average all firms that enter a market are the same, then $\pi_{ni} = J_{ni}/J_n$, and $J_n = J_{ni}/\pi_{ni}$, which will be used later.

Ricardian model. If there are no fixed costs and $J = 1$ then taking $c_n \to \infty$ in equation (2) and assuming $\alpha = 1$ yields the equation for P_n in EK 2002 (using $\overline{m} = 1$).

Monopolistic competition. Now consider $J \to \infty$. To see the implications of this, note that EKK-9 can be written as

$$\widetilde{P}_n = E_\eta \left[E[\alpha \mid \eta] J^{1-1/\widetilde{\theta}} \int_0^{\left(\eta x_n / \widetilde{P}_n\right)^{\widetilde{\theta}} / J} t^{-1/\widetilde{\theta}} e^{-t} dt \right]$$

But applying L'Hopital's rule we get that

$$
\begin{aligned}
& \lim_{J\to\infty} J^{1-1/\widetilde{\theta}} \int_0^{\left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}}/J} t^{-1/\widetilde{\theta}} e^{-t} dt \\
= & \lim_{J\to\infty} \frac{-\left(\eta x_n/\widetilde{P}_n\right)^{-1} J^{1/\widetilde{\theta}} e^{-\left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}}/J} \left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}} J^{-2}}{-\left(1-1/\widetilde{\theta}\right) J^{-\left(1-1/\widetilde{\theta}\right)-1}} \\
= & \lim_{J\to\infty} \frac{e^{-\left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}}/J} \left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}-1}}{1-1/\widetilde{\theta}} = \frac{\left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}-1}}{1-1/\widetilde{\theta}}
\end{aligned}
$$

Plugging this above we get

$$\widetilde{P}_n = E_\eta \left[E[\alpha \mid \eta] \frac{\left(\eta x_n / \widetilde{P}_n\right)^{\widetilde{\theta}-1}}{1 - 1/\widetilde{\theta}} \right] = a_1 \frac{\left(x_n / \widetilde{P}_n\right)^{\widetilde{\theta}-1}}{1 - 1/\widetilde{\theta}}$$

where

$$a_1 \equiv E_\eta \left[E\left[\alpha \mid \eta\right] \eta^{\widetilde{\theta}-1} \right]$$

This implies

$$\widetilde{P}_n = \left(1 - 1/\widetilde{\theta}\right)^{-1/\widetilde{\theta}} a_1^{1/\widetilde{\theta}} x_n^{1-1/\widetilde{\theta}}$$

Finally, using $P_n = \bar{m} \widetilde{P}_n^{-1/(\sigma-1)} \phi_n^{-1/\theta}$, and noting that $1/\widetilde{\theta}(\sigma - 1) = 1/\theta$, we get EKK-12, or

$$P_n = \bar{m} \left(1 - 1/\widetilde{\theta}\right)^{1/\theta} a_1^{-1/\theta} x_n^{-(1-1/\widetilde{\theta})/(\sigma-1)} \phi_n^{-1/\theta}$$

The cutoff $\bar{c}_n$ can now be obtained from $\bar{c}_n = x_n^{1/(\sigma-1)} P_n/\bar{m}$, so using EKK-12 we get

$$\begin{aligned} \bar{c}_n &= x_n^{1/(\sigma-1)} \left(1 - 1/\widetilde{\theta}\right)^{1/\theta} a_1^{-1/\theta} x_n^{-(1-1/\widetilde{\theta})/(\sigma-1)} \phi_n^{-1/\theta} \\ &= \left(1 - 1/\widetilde{\theta}\right)^{1/\theta} \left(\frac{a_1 \phi_n}{x_n}\right)^{-1/\theta} \end{aligned}$$

Note: I think there is a typo in EKK-13.

The measure of entrants with cost less than or equal to c can be obtained from

$$J\left\{1-\exp\left[-(T_i/J)z^{-\theta}\right]\right\}$$

by taking the limit as $J\longrightarrow\infty$, to get (applying L'Hopital's rule)

$$\mu_n^{\infty}(c)\equiv\lim_{J\to\infty}\mu_n(c)=\phi_n c^{\theta}$$

To obtain the measure of entrants in market n in equilibrium, we can do it two ways. First, one can simply use $E_\eta\mu_n^{\infty}(\eta^{1/(\sigma-1)}\bar{c}_n)$, which yields

$$\begin{aligned}J_n &= E_\eta\phi_n\eta^{\widetilde{\theta}}\left(1-1/\widetilde{\theta}\right)\left(\frac{a_1\phi_n}{x_n}\right)^{-1}\\ &= \left(1-1/\widetilde{\theta}\right)x_nE_\eta\left(\eta^{\widetilde{\theta}}\right)/a_1\end{aligned}$$

Note: I think there is a typo in EKK-15.

To check this result, one can apply a slightly different procedure and take $J \longrightarrow \infty$ in the expression for J_n (equation 4 above or EKK-10) to get, applying L'Hopital's rule,

$$\begin{aligned}
& \lim_{J\to\infty} \frac{1 - \int e^{-\left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}}/J} df(\eta)}{J^{-1}} \\
= \; & \lim_{J\to\infty} \frac{-\int \left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}} J^{-2} e^{-\left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}}/J} df(\eta)}{-J^{-2}} \\
= \; & \lim_{J\to\infty} \int \left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}} e^{-\left(\eta x_n/\widetilde{P}_n\right)^{\widetilde{\theta}}/J} df(\eta) \\
= \; & \left(x_n/\widetilde{P}_n\right)^{\widetilde{\theta}} \int \eta^{\widetilde{\theta}} df(\eta) \\
= \; & \left(x_n/\widetilde{P}_n\right)^{\widetilde{\theta}} E(\eta^{\widetilde{\theta}})
\end{aligned}$$

But from

$$\widetilde{P}_n = \left(1 - 1/\widetilde{\theta}\right)^{-1/\widetilde{\theta}} a_1^{1/\widetilde{\theta}} x_n^{1-1/\widetilde{\theta}}$$

we see that

$$\begin{aligned} \left(x_n/\widetilde{P}_n\right)^{\widetilde{\theta}} &= x_n^{\widetilde{\theta}} \left(1 - 1/\widetilde{\theta}\right) a_1^{-1} x_n^{1-1/\widetilde{\theta}} \\ &= \left(1 - 1/\widetilde{\theta}\right) x_n/a_1 \end{aligned}$$

and hence we finally get that

$$J_n = \left(1 - 1/\widetilde{\theta}\right) x_n E(\eta^{\widetilde{\theta}})/a_1$$

A special case with no preference or fixed cost shocks. This entails $\alpha_n(j) = \varepsilon_n(j) = 1$ for all n, j. Define $\bar{c}_{ni} = \bar{c}_n / d_{ni}$ and rank $\bar{c}_{ni}$ across n as

$$\bar{c}_i^{(1)} \geq \bar{c}_i^{(2)} \geq ... \geq \bar{c}_i^{(k)} \geq ... \geq \bar{c}_i^{(N)}$$

Any firm that sells in $k-th$ ranked market has a domestic cost c below $\bar{c}_i^{(k)}$ which is also below $\bar{c}_i^{(k')}$ for all $k' < k$. This implies a hiearchy of markets.

The sales distribution. Let $F_n(x) = 1 - \Pr\left[X \geq x \mid X \geq \sigma E_n\right]$. We know that $X = (mc/P_n)^{1-\sigma} X_n$, hence $X \geq x$ is equivalent to

$$\begin{aligned} (\bar{m}c/P_n)^{1-\sigma} X_n &\geq x \\ X_n / x &\geq (\bar{m}c/P_n)^{\sigma-1} \\ c &\leq (X_n/x)^{1/(\sigma-1)} P_n/\bar{m} \end{aligned}$$

Thus (note typos in EKK),

$$F_n(x) = 1 - \Pr\left[c \leq (X_n/x)^{1/(\sigma-1)} P_n/\bar{m} \mid c \leq (X_n/\sigma E_n)^{1/(\sigma-1)} P_n/\bar{m}\right]$$

But we know that the measure of entrants with cost less than or equal to c is $\phi_n c^\theta$, thus for $x \geq \sigma E_n$ we have

$$\begin{aligned} F_n(x) &= 1 - \frac{\phi_n \left(X_n/x\right)^{\widetilde{\theta}} \left(P_n/\bar{m}\right)^\theta}{\phi_n \left(X_n/\sigma E_n\right)^{\widetilde{\theta}} \left(P_n/\bar{m}\right)^\theta} \\ &= 1 - \left(\frac{x}{\sigma E_n}\right)^{-\widetilde{\theta}} \end{aligned}$$

which is a Pareto distribution starting at σE_n with shape parameter $\widetilde{\theta}$.

This implies that means sales are

$$\bar{x}_n = \frac{\sigma E_n}{1 - 1/\widetilde{\theta}} \tag{5}$$

Entry. The measure of firms selling in market n is then

$$J_n = X_n/\bar{x}_n = \left(1 - 1/\widetilde{\theta}\right) x_n$$

Sales in a Market and Number of Markets Served. Let $J_{ni}^{(k)}$ be the measure of firms from i selling in market n that also sell in at least k less popular markets than n, and let $\underline{x}_{ni}^{(k)}$ be the minimum sales in market n of those firms.

From the sales distribution above we know that

$$J_{ni}^{(k)} = J_{ni}^{(0)} \left(\frac{\underline{x}_{ni}^{(k)}}{\sigma E_n} \right)^{-\widetilde{\theta}}$$

and hence

$$\underline{x}_{ni}^{(k)} = \sigma E_n \left(\frac{J_{ni}^{(k)}}{J_{ni}^{(0)}} \right)^{-1/\widetilde{\theta}}$$

This implies that the mean sales in market n of firms from i selling to at least k less popular destinations than n is $\underline{x}_{ni}^{(k)} / \left(1 - 1/\widetilde{\theta}\right)$, or EKK-18:

$$\bar{x}_{ni}^{(k)} = \frac{\sigma E_n}{\left(1 - 1/\widetilde{\theta}\right)} \left(\frac{J_{ni}^{(k)}}{J_{ni}^{(0)}} \right)^{-1/\widetilde{\theta}}$$

This is a precise relationship between a firm's sales in any given market and the number of less popular markets it sells in.

Focusing on French firms, EKK note that for a firm to sell in market n, it has to pass to hurdles.

First, the *entry hurdle*, which entails

$$X_n^*(j) = \alpha_n(j) \left(\frac{\overline{m} c_{nF}(j)}{P_n} \right)^{1-\sigma} X_n \geq \sigma E_n \varepsilon_n(j)$$

where $c_{nF}(j) = w_F d_{nF}/z_F(j)$.

Second, the *competition hurdle*, which is

$$c_{nF}(j) < \min_{i \neq F} \{c_{ni}(j)\}$$

Note that a firm can pass the competition hurdle and still not enter if profits are negative. Similarly, a firm can pass the entry hurdle but not enter because another firm has a lower cost for that good.

Quantification I: Monopolistic Competition with no Shocks

The hierarchy of markets is violated because there are firms selling in the k-th most popular market that do not sell to all the first k markets.

For example, many exporters do not export to Belgium, which is the most popular destination for French exports. But this works roughly - Figure 1. So it makes sense to check the other implications of the model.

Figures 2a plots the following relationship

$$\bar{x}_{ni}^{(k)} = \frac{\sigma E_n}{\left(1 - 1/\widetilde{\theta}\right)} \left(\frac{J_{ni}^{(k)}}{J_{ni}^{(0)}}\right)^{-1/\widetilde{\theta}}$$

in logs and for $n = F$. It is linear, as it should be, and allows an estimation of $\widetilde{\theta} = 1.5$. Figure 2b plots the same thing but for the measure of firms selling to the k-th most popular destination.

Recall that $\bar{x}_n = \sigma E_n/(1 - 1/\widetilde{\theta})$ is the average sales in market n, and that $\bar{x}_{nF} = \bar{x}_n$. Thus, EKK use

$$(1 - 1/\widetilde{\theta})\bar{x}_{nF}$$

with $\bar{x}_{nF} = X_{nF}/J_{nF}$ as a way to estimate σE_n.

This is plotted in Figure 3 against total market size X_n (manufacturing absorption) on a log scale.

The relationship is linear, with slope of 0.36, which suggests that larger markets have higher entry costs.

Recall that

$$P_n = \bar{m}\left(1 - 1/\widetilde{\theta}\right)^{1/\theta} a_1^{-1/\theta} x_n^{-(1-1/\widetilde{\theta})/(\sigma-1)} \phi_n^{-1/\theta}$$

This establishes a relationship between the price index and adjusted market size, $x_n = X_n/\sigma E_n$. Using $(1 - 1/\widetilde{\theta})\bar{x}_{nF} = \sigma E_n$, and $\widetilde{\theta} = 1.5$, the component related to x_n is

$$\left(\frac{X_n}{(1 - 1/\widetilde{\theta})\bar{x}_{nF}}\right)^{-1/3(\sigma-1)}$$

A missing Figure (not Figure 4) plots this against market size X_n on a logarithmic scale. Presumably it shows that larger markets have lower prices thanks to more variety.

This effect is attenuated by heterogeneity. With $\sigma =?$ the elasticity is -0.046, while with no heterogeneity ($\theta < \infty$) this would be $-1/(\sigma - 1)$, which is 3 times larger.

The distribution of sales in any given market is given by

$$F_n(x) = p = 1 - \left(\frac{x}{\sigma E_n}\right)^{-1.5} \quad \text{for} \quad x \geq \sigma E_n$$

Figure 4 plots

$$1 - p = \left(\frac{x}{\sigma E_n}\right)^{-1.5}$$

in logs (averaged for three groups of countries according to whether French firms are small, medium or large).

It should be linear, but this is violated at the lower end of the distribution. Instead of ending at $x = \sigma E_n$ it appears to continue towards smaller sales.

Moreover, the slope at upper end is too step, with a slope closer to -1 rather than $-2/3$ - so model understates "curvature" in sales across firms.