Equilibrium vs Optimum in a simple version of the Dixit Stiglitz model

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1 Set-up

Assume CD preferences for homogenous good z and composite good X, with share of expenditure on z equal to α. X is assembled from CES with σ. The homogenous good and all varieties have constant marginal cost equal to one. There is a fixed cost F (in units of labor) for each variety of the differentiated good. There is one unit of labor.

2 Optimum

Letting $\rho \equiv (1 - 1/\sigma)$ (i.e., the inverse of the mark-up) then Dixit Stiglitz (DS) find that the (unconstrained) optimum n satisfies (equation 28, using $s = 1-\alpha$):

$$\frac{1-\alpha}{n} = \frac{F\rho/(1-\rho)}{1-Fn}$$

This implies that

$$nF = \frac{1-\alpha}{\sigma-\alpha}$$

Moreover, DS show that in the optimum the quantity produced of each differentiated good, x, is given by

$$x = (\sigma - 1)F$$

Finally, from the full-employment condition, $z + n(x + F) = 1$, we get $z = 1 - \sigma nF$, which implies

$$z = \frac{\alpha(\sigma-1)}{\sigma-\alpha}$$

3 Case $\alpha = 0$

With $\alpha = 0$, so that the z good disappears, then

$$nF = 1/\sigma$$

and hence $n = (1/\sigma F)$. This is exactly what is obtained for n from the equilibrium for the case with $\alpha = 0$. This follows from the fact that, for that equilibrium, the zero profit condition implies $x = F(\sigma - 1)$, and plugging into the full employment condition, we get $n(F + x) = n\sigma F = 1$. In fact, using $s = 1$ one can immediately check from DS that the equilibrium and (unconstrained) optimum variety are the same.

Why are the equilibrium and optimum the same? How is it that the equilbrium achieves the first best allocation? This question is relevant because of the existence of two pecuniary externalities: the consumer surplus externality (entrants don't take into account the consumer surplus they generate), and the business stealing externality (entrants do not take into account the reduction in profits to other firms). What happens is that if $\alpha = 0$ then the two effects exactly cancel each other.

4 Case $\alpha > 0$

When $\alpha > 0$, I will show that the equilibrium when there is a consumption subsidy equal to ρ replicates the optimum described above. Given such a subsidy, consumers would pay nx for X, whereas producers would get nx/ρ, hence the cost of the subsidy (and hence lump sum tax to consumers) is $(1/\rho - 1)nx$ and hence consumer income is

$$I = 1 - (1/\rho - 1)nx$$

But consumers must spend $(1 - \alpha)I$ on the X good, hence $nx = (1 - \alpha)I$, and $I = 1 - (1/\rho - 1)(1 - \alpha)I$, or

$$I = \frac{1}{1 + (1/\rho - 1)(1 - \alpha)}$$

Thus, from $z = \alpha I$ we get

$$z = \frac{\alpha(\sigma - 1)}{\sigma - \alpha}$$

which is the same as in the optimum. Moreover, x must be the same in equilibrium as in the optimum because the zero profit condition is

$$(1/\rho - 1)x = F$$

or $x = (\sigma - 1)F$. Finally, from the full employment condition we can then get that n must also be the same as in the optimum.