1 Generalizations from $2x2$ to MxN

In the $2x2$ (two factor, two good) model we discussed several results, in particular FPI, FPE, Stolper-Samuelson (SS) and Rybczynski (R). It is natural to ask how these results generalize to the case of M factors and N goods.

1.1 FPE

Feenstra (chapter 3) explores FPI and FPE in the NxN case by considering a generalization of the *no FIR* condition that we discussed for the $2x2$ case, and then argues that there is FPE in a two-country world if there is full diversification in both countries.

I find this discussion to be less instructive than what is done in Dixit and Norman (chapter 4, section 4). They start from an integrated equilibrium and then consider the "size" of the set of the partions of worldwide factor endowments among the two (or more) countries in the world. It is shown that this set has full dimensionality if $N \geq M$ and the technologies of M goods are linearly independent. Thus, if $N < M$ then the set of factor partitions consistent with FPE has zero dimensionality.

Note: when there are non-tradable goods, what matters is the number of *tradable goods*. This is because each non-tradable good also comes with an extra condition (i.e., output equal demand in each country), so this reduces dimensionality to the set spanned by the "technology vectors," or A^i vectors.

Note: if $N > M$ then the FPE set has full dimensionality (as long as the technologies of M goods are linearly independent), but since necessarily there are some goods whose technologies are linearly dependent ($N - M > 0$), then there is indeterminacy. Still, the factor content of trade is well determined, as in the HOV model.

Question: with $N > M$, the equilibrium entails N equations in M unknowns. Does this mean that diversification (producing all N goods) is unlikely?

1.2 Generalizations of SS and R for NxN

In the NxN case the SS and R theorems generalize nicely, although in weaker form.

The SS theorem now says that for a change in the price of each good there will exist some factor that gains and some factor that loses in real terms. Formally, for each $i \ \exists \ j$ and k $s.t.$

$$\hat{w}_j > \hat{p}_i > 0 > \hat{w}_k \quad (\hat{p}_l = 0 \text{ for all } l \neq i)$$

where $\hat{x} = dx/x$. This result can be understood through reasoning by contradiction. Imagine that all factor prices increased as much as p_i; then the zero profit condition would still hold for i, but there would be negative profits in all the other goods. Similarly, if factor prices all increased by less than p_i then there would be positive profits in i. Thus, it must be that at least one factor price increases by more than p_i, and some other factor price falls (otherwise the cost of all other goods would increase, again leading to negative profits).

The R theorem now says that for an increase in the endowment of each factor, there exists some good whose output increases and another whose output decreases. The intuition is similar to the one for SS. See Feenstra.

Note: the SS as stated above doesn't say that each factor suffers from the increase in the price of some good (as is the case in the $2x2$ case). But Jones and Sheinkman (1977) show that in fact we can say this. Basically, this holds because of what Samuelson called the "reciprocity relations," namely that $dw_j/dp_i = dy_i/dV_j$. So "every factor has a good that is a natural enemy, in the sense that raising the price of that good will lower the real return to the factor" (Feenstra, citing Jones and Sheinkman).

1.3 SS and R for $M > M$

There are no general results for SS and R in this case, except for the Specific Factors Model. See separate note.

1.4 Case $N > M$

Recall that in this case the FPE set has full dimensionality (as long as M technologies are linearly independent). But the flip side of this is that output levels are not determined. Thus, under FPE there is no way to do comparative statics as in SS or R!

There are basically three ways to get away from indeterminacy when $N > M$: transportation costs or trade barriers, productivity differences (Ricardian - HO model of Davis), and being outside the FPE set. We will now focus on this later case.