Lectures Notes on the Hecksher-Ohlin Theory in the Short, Medium and the Long Run

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The HO model is a weird combination of short and long run features. On the one hand, K is fixed, which is a short run assumption, but on the other hand K is perfectly mobile across sectors, a long run assumption. Because of this, sometimes we say that HO is a medium run model, but this is clearly not satisfactory. These notes discuss this issue at length.

1 From the short to the "medium" run

There is a long tradition of thinking of the specific factors model as the short-run version of the HO model. In "steady state" capital and labor are allocated according to the HO model, with the rate of return equalized across the two sectors. After a price shock there is an initial adjustment as described in the specific factors model, but this leads to a difference in returns to capital across the two sectors, which generates an adjustment process to the new steady state. This adjustment process is analyzed graphically by Neary (1978), but without determining the full adjustment process since it is not clear what prevents capital from switching sectors immediately.

One can also think about this in terms of the short run and medium run PPF curves. The medium run PPF curve is the one we know from the HO model. For each point on this medium run PPF curve, there is a corresponding short-run PPF curve which is tangent to the medium run curve, but more concave. In other words, the medium run PPF curve is the envelope of all the short run curves as we vary the capital allocation between the two sectors. The short run PPF curve associated with point x on the envelop or medium run PPF curve is the specific factors PPF curve with the capital stock allocated according to the HO allocation at point x.

Matsuyama (1992) has a more complete analysis of the adjustment process. In his model there is only labor, but there is heterogeneity, so that some workers are better in one sector than the other. Thus, the PPF is concave and there is an interior solution given international prices. There is a constant stock of workers, but with overlapping generations: in continous time there is a constant probability of death p equal to the rate at which new workers are "born." New workers decide what sector to join and this decision is permanent: there is no switching sectors afterwards. Thus, workers are "putty-clay" - they are born as putty, and once they join a sector, they become clay and cannot be reshaped to fit the other sector. Matsuyama then analyzes the transition path as the economy goes from one steady state point on the PPF to another. This transition takes place as new workers choose the sector whose price has increased.

One would like to have something like this, but for capital. There would be a crucial difference in that the stock of capital would be endogenous. I don't know of a paper that does this... for now, let's consider the alternative case in which the stock of capital is always perfectly mobile, but fixed in the "medium run" and endogenous in the long run. Thus, we move from the analysis of the short and medium run (Neary, Matsuyama), to an analysis of the medium and long run (Stiglitz 1970, Findlay 1995, Atkeson and Kehoe, Ventura, Cavalcanti-Ferreira and Trejos, etc.)

2 From the "medium" to the long run

Consider an economy with two factors, capital and labor, and two tradable intermediate goods that are used to produce a non-tradable final good that can be either consumed or invested. Assume to simplify that there is no depreciation and no population growth. The intermediate goods are produced with CRS, and there is no FIR, with good 1 being labor intensive. There is a bunch of identical economies, differing perhaps in their initial stock of capital, and in each economy there is a representative consumer with logarithmic utility and discount rate ρ, so that $U_t = \int_t^\infty \ln(c_s) e^{-\rho(s-t)} ds$.

In steady state, with no endogenous growth, the rate of return to capital must be equal to the discount rate, $R = \rho$. Thus, even with no trade, we would have FPE (same R implies same w given the same technology). If countries reach their steady state before they start trading, then there would be no trade. But this is not the only possibility.

Let the final good serve as numeraire. Given the technology to produce output from the intermediate goods, there is a set of prices of these intermediate goods such that the zero profit condition is satisfied for the production of output, $1 = C(p_1, p_2)$. By Stolper Samuelson, each of these price pairs is associated with a pair of factor prices, (r, w). The equilibrium $(\widetilde{p}_1, \widetilde{p}_2)$ is the one that generates a return on capital equal to the discount rate, $r = \rho$. We know that this also determines the wage, w.

For the equilibrium $(\widetilde{p}_1, \widetilde{p}_2)$ we know that the Rybczynski line of capital expansion (which unites the tangency of all PPF curves with same L and different levels of K) has $r = \rho$ at all its points. It also has the same wage, w. This Rybczynski line can be obtained simply from the full employment condition for labor, which is obtained once we know the unit labor requirements for both intermediate goods, which we determine from the respective technologies given that we know (r, w).

From cost minimization in the production of output from intermediate goods, we can obtain a relative demand for intermediate goods, so that $x_2/x_1 = \lambda^*(p_1/p_2)$. The intersection of the line $x_2/x_1 = \lambda^*(\widetilde{p}_1/\widetilde{p}_2)$ with the Rybzynski line (the FF' curve) gives the equilibrium x_2/x_1 in the closed economy.

Now let's move to the open economy. With free trade, prices of intermediate goods are equalized and we must have FPE in steady state because both countries must have $r = \rho$. Thus, at any point the world equilibrium is given by the equilibrium of the integrated economy, which is the one we derived above for the closed economy. Thus, both countries must be on the same FF' curve derived above.

The world's relative demand for intermediate goods is $x_2/x_1 = \lambda^*(p_1/p_2)$ so the world's production of intermediate goods must satisfy $x_2/x_1 = \lambda^*(\widetilde{p}_1/\widetilde{p}_2)$. This case is analyzed in Findlay (Factor Proportions, Trade and Growth, MIT Press, 1995).

The world's production of intermediate goods can satisfy this restriction in many different ways, thus there is multiplicity of steady state equilibria. As long as both countries are on the FF' line and the joint or world's production of intermediate goods satisfies the above condition, then we have an equilibrium

Homework: prove that transportation costs eliminate this multiplicity, and only one equilibrium survives, namely that one where both countries produce intermediate goods with the same ratio $x_2/x_1 = \lambda^*(\widetilde{p}_1/\widetilde{p}_2)$. So is the HO model non robust to capital accumulation and small transportation costs? We need another source of heterogeneity across countries...

3 A simple model with the three runs

There is a small economy, two factors (capital and labor), and two intermediate goods which produce a final output that can be either consumed or invested. Capital is putty-clay. Production of intermediate good i is $Y_i = A_i X_i$, where $X_i = \gamma K_i^{\alpha} L_i^{1-\alpha}$ and $\gamma \equiv \alpha^{\alpha}(1-\alpha)^{1-\alpha}$. Note that there are no differences in factor intensities across sectors. Also note that the unit cost of X is simply $R^{\alpha} w^{1-\alpha}$, where R is the rental cost of capital. Output is produced according to $(\beta^{\beta}(1-\beta)^{1-\beta}) Y_1^{\beta} Y_2^{1-\beta}$, where it is assumed that $\phi = \beta^{\beta}(1-\beta)^{1-\beta}$ so that the unit cost of output is simply $p_1^{\beta} p_2^{1-\beta}$. We assume that international prices are given by $p_1 = p_2 = 1$, and that the price of output is also one. This would be the case as long as $A_1 = A_2$ in the rest of the world.

In the small economy, we assume that $A_2 = 1$ and $A_1 < 1$. From now on we use A to denote A_1. Given this assumption, the small economy, which from now on we refer to as Home, has comparative advantage in good 2, and hence specializes completely in that sector. Letting ρ be the intertemporal rate of discount and δ the rate of depreciation, then in steady state must have $R = \rho + \delta$, and also

$$R^{\alpha} w^{1-\alpha} = 1$$

together with

$$(1-\alpha)\gamma K^{\alpha}L^{-\alpha}=w$$

This determines the wage and the capital stock in Home.

Now imagine that a tariff $\tau > 1/A$ is imposed. Assume that this is a surprise, so that capital was originally all allocated to sector 1. Also, assume that there is perfect mobility of capital across countries. I now show that the adjustment process entails a discrete jump in K and then an adjustment downwards so that in the new steady state the capital stock is lower than under free trade. Moreover, there is no trade in the new steady state.

In a first phase of the adjustment process, the economy will continue to export intermediate good 2 simply because it has a high level of K_2. During this phase, K_2 declines at the rate of depreciation, while K_1 first jumps and then continues to increase according to the reallocation of labor from sector 2 to sector 1. In turn, this reallocation of labor is completely determined by the decline in K_2, which releases labor in that sector that can be reallocated to sector 1. At some point, K_2 will have fallen so much that the economy will no longer trade. At this point we get a second adjustment phase, where K_2 continues to fall but prices are determined by domestic preferences.

In the first phase of adjustment, domestic prices are determined by the international prices and the tariff, so that $p_1 = \tau$ and $p_2 = 1$. At the original (free trade) wage, the unit cost of good 1 becomes lower than its domestic price, since

$$(1/A)R^{\alpha}w^{1-\alpha} < \tau$$

at the original wage. Hence, in the transition the wage must jump to equalize this relationship. This pins down the wage in the first phase of the transition, where the economy continues to trade given its large quantity of K_2. But given a quantity of capital K_i in sector i, the condition that the value of the marginal product of labor be equal to the wage implies

$$(1-\alpha)\gamma K_2^{\alpha}L_2^{-\alpha}=w \tag{1}$$

and

$$\tau(1-\alpha)\gamma A K_1^{\alpha}L_1^{-\alpha}=w \tag{2}$$

Together with

$$(1/A)R^{\alpha}w^{1-\alpha}=\tau \tag{3}$$

and the full employment condition $L_1 + L_2 = L$, this forms a system of four equations in four unknowns, K_1, L_1, L_2, and w. Equation 3 determines the wage; plugging the wage into equation 1 determines L_2; and plugging the result into equation 2 we obtain K_1.

What happens to total capital on impact? Note that since w increases then L_2 decreases and this implies that the MPK falls below R in sector 2, which triggers a process of declining K_2 according to the rate of depreciation. The decline in L_2 releases some labor for sector 1, which then leads to a positive K_1, implying that necessarily total capital increases on impact.

After the jump in the capital stock, the capital stock will adjust downwards. To see this, note that

$$(K_2/L_2)^\alpha = \frac{w}{(1-\alpha)\gamma} > \frac{w}{(1-\alpha)\gamma\tau A} = (K_1/L_1)^\alpha$$

Thus, since

$$K/L = (K_1/L_1)(L_1/L) + (K_2/L_2)(L_2/L)$$

then as sector 2 contracts K/L falls, which implies that K falls in the transition during the first phase. In summary, K jumps and then falls. This process continues until a point at which there is no trade. From this point onwards, as K_2 continues to fall, domestic prices are such that there are no incentives to trade, and the analysis is one of convergence to the autarky steady state.

Let's first determine the new autarky steady state. Using output as the numeraire, then we know that prices of goods 1 and 2 must satisfy

$$p_1^\beta p_2^{1-\beta} = 1 \tag{4}$$

But we know that p_i must be equal to the unit cost in sector i, hence we have the following two equations

$$p_2 = R^\alpha w^{1-\alpha}$$

and

$$p_1 = (1/A) R^\alpha w^{1-\alpha} \tag{5}$$

Plugging this in the equation above, we get

$$R^\alpha w^{1-\alpha} = A^\beta \tag{6}$$

This determines the wage. Moreover, from the requirement that the value of the MPL equal the price, we can get

$$(K_2/L_2)^\alpha = \frac{w}{p_2(1-\alpha)\gamma} \tag{7}$$

and

$$(K_1/L_1)^\alpha = \frac{w}{p_1(1-\alpha)\gamma A} \tag{8}$$

which then implies that the capital labor ratio is the same in both sectors, as one should expect. Thus, the total capital stock is obtained simply by solving for w from 6, plugging into equation 7 and then solving for K by using $K = kL$, where k is the capital-labor ratio that results from this procedure.

How does the new (autarky) steady state K compare with the open economy K? We need to compare the new capital-labor ratio with the original one, but this just entails comparing w/p_2 under autarky with w under free trade. To do this comparison, note that under autarky we have

$$\begin{aligned} \frac{w}{p_2} &= \frac{w}{R^\alpha w^{1-\alpha}} = \frac{w^\alpha}{R^\alpha} = \frac{\left[A^\beta/R^\alpha\right]^{\alpha/(1-\alpha)}}{R^\alpha} \\ &= A^{\alpha\beta/(1-\alpha)} R^{-\alpha(1+\alpha/(1-\alpha))} = A^{\alpha\beta/(1-\alpha)} R^{\alpha/(\alpha-1)} \end{aligned}$$

whereas under free trade

$$w = (1/R^{\alpha})^{1/(1-\alpha)} = R^{\alpha/(\alpha-1)}$$

Thus, since $A < 1$ then w/p_2 under autarky is lower than w.

Finally, let's explore what happens in the second phase of adjustment, where the economy is not trading, but before reaching the new steady state characterized above. Here the equilibrium must satisfy equations 7 and 8 together with 5 and 4. Moreover, we must also have

$$\frac{p_1 Y_1}{p_2 Y_2} = \frac{\beta}{1-\beta}$$

but this implies

$$\frac{p_1 Y_1}{p_2 Y_2} = \frac{p_1 A \gamma K_1^{\alpha} L_1^{1-\alpha}}{p_2 \gamma K_2^{\alpha} L_2^{1-\alpha}} = \left(\frac{p_1 A \gamma K_1^{\alpha} L_1^{-\alpha}}{p_2 \gamma K_2^{\alpha} L_2^{-\alpha}}\right)\left(\frac{L_1}{L_2}\right) = \frac{L_1}{L_2} = \frac{\beta}{1-\beta}$$

Since L is fixed, then this implies that in the second phase of adjustment the labor allocation is fixed. Thus, given these fixed levels of L_1 and L_2 in the second phase, we have a system of four equations (7, 8, 5 and 4) in four unknowns (K_1, p_1, p_2, w), with K_2 determined by the rate of depreciation. Note that K_1 appears only in equation 8, so we can focus on the system composed of equations 7, 5 and 4, in the three unknowns p_1, p_2, and w.

It is interesting to explore what happens to K_1 in this phase. To do so, from 8 we need to know what happens to w/p_1. But from 5 we have

$$w/p_1 = A R^{-\alpha} w^{\alpha} \tag{9}$$

so all we need to know is what happens to w as K_2 falls in this phase. To do this, first note that 4 implies $p_1 = p_2^{-\phi}$, where $\phi \equiv (1-\beta)/\beta$. Plugging this into 5 we get $A p_2^{-\phi} = R^{\alpha} w^{1-\alpha}$or

$$A(w/p_2)^{\phi} = R^{\alpha} w^{1-\alpha+\phi}$$

Now, as K_2 falls we know from 7 that w/p_2 decreases, and from the previous equation we see that this leads to a decline in w. This implies that the wage will be decreasing in this second phase of adjustment, which from equation 9 implies that w/p_1 will also be decreasing. Finally, from 8 we see that K_1 will be decreasing, which implies that $K_1 + K_2$ in the second phase of adjustment will be falling.