Why was Leontief Wrong?

Leamer showed that even if the HOV model is right, one could still get the Leontief paradox. Thus, it is no paradox at all.

To see this, first read the Leamer Theorem in Feenstra, page 39. This theorem can be applied to two factors of production even in a setting that is more general than the simple 2x2x2 model. Thus, this theorem gives the right tool to do the kind of empirical analysis that Leontief wanted to do. More interestingly, when Leamer did the Leontief exercise again in this way, he found that the data was consistent with the HOV model: the capital/labor ratio in production in the US was indeed higher than the corresponding ratio in consumption in the US, as we would expect from HOV if capital was abundant relative to labor in the US (in the sense of Leamer).

This short note explores the reason why the Leontief exercise is not the right one. Let's first show that if there is trade balance in L and K, then the Leontief test would have been fine as a way to test the HOV theory. That is, if capital is abundant relative to labor in the US according to the Leamer definition, then it would also be the case (if the HOV theory is right) that the capital-labor ratio in exports is higher than the capital-labor ratio in imports.

To see this, let Lp and Lc be labor in production and consumption, respectively, and similarly with Kp and Kc. If capital is abundant relative to labor in the US in the sense of Leamer, this implies from Leamer's theorem that:

$$Kp/Lp > Kc/Lc$$

But this implies that:

(1) $$Kp/Kc > Lp/Lc$$

Note that if there is trade balance in L and K, then it must be that $w^*(Lp-Lc) + r^*(Kp-Kc) = 0$. From (1) we know that if $Kp/Kc \leq 1$, then also $Lp/Lc < 1$. But this would imply that $Kp \leq Kc$ and $Lp < Lc$, so there could not be trade balance. Thus it cannot be true that $Kp/Kc \leq 1$ → we have proven that $Kp/Kc > 1$, or $Kp > Kc$. From trade balance this implies that $Lp < Lc$, or $Lp/Lc < 1$. Thus, we have that if capital is abundant relative to labor in the US in the sense of Leamer, and if there is trade balance in L and K, then:

$$Kp/Kc > 1 > Lp/Lc$$

This implies that

(2) $$Kp - Kc > 0 > Lp - Lc$$

Now let Lx and Lm be the labor embodied in exports and imports, respectively, and similarly with Kx and Km. Then:

$$Kp - Kc = Kx - Km$$
$$Lp - Lc = Lx - Lm$$

Thus, from (2) we have that:

$$Kx > Km$$
$$Lx < Lm$$

This implies that:

$$Kx/Lx > Km/Lm$$

Thus, under trade balance in K and L, and assuming that capital relative to labor in the US in the Leamer sense, then HOV implies that the capital-labor ratio in exports is higher than the capital-labor ratio in imports. In this case, the Leontief exercise would be correct, in that finding the opposite (as Leontief did) would indeed be inconsistent with the model. The problem is that Leontief did not check that trade was balanced, and it wasn't!

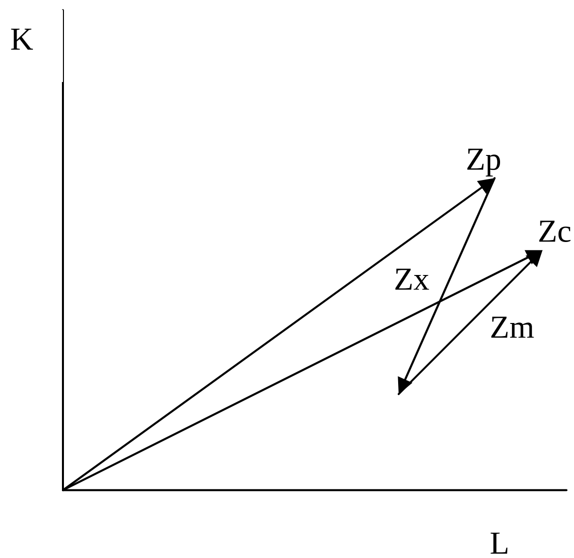
To see what can happen if there is no trade balance, consider the case where $Kp > Kc$ and $Lp > Lc$. If capital is abundant relative to labor in the US, then the capital-labor ratio in production must be higher than the capital-labor ratio in consumption in the US (from Leamer Theorem, which doesn't depend on trade balance). Imagine that $Kp/Lp = 2$ and $Kc/Lc = 1$. Imagine also that $Kp = 200$ and $Lp = 100$, and $Kc = Lc = 50$.

Consider the following possibility: $Kx = 175$ and $Lx = 55$. Then it must be that $Km = 25$ and $Lm = 5$. But this implies that $Kx/Lx = 170/55 = 3.18$ and $Km/Lm = 5$. This is the Leontief "paradox."

To see why this can happen, draw a graph with K on the vertical axis and L on the horizontal axis. Draw a line (ray P) from the origin with the slope given by the capital-labor ratio in production and another line (ray C) from the origin with slope given by the capital-labor ratio in consumption. To go from point $Zp = (Lp,Kp)$ in ray P to point $Zc = (Lc,Kc)$ you have to substract vector $Zx = (Lx,Kx)$ from Zp and then add vector $Zm = (Lm,Km)$.

If there is trade balance, then Zp will have more capital and less labor than the Zc (Zp will be to the North-West of Zc). You can check that in this case there is no way to have vectors Zx and Zm with $Kx/Lx < Km/Lm$ such that $Zp - Zx + Zm = Zc$. See the graph below, on the left.

On the other hand, if there is trade surplus, so that $Lp > Lc$ and $Kp > Kc$, then *it is* possible to find vectors Zx and Zm with $Kx/Lx < Km/Lm$ such that $Zp - Zx + Zm = Zc$. This is possible because Zc is to the South-West of Zp. See the graph below, on the right.


K
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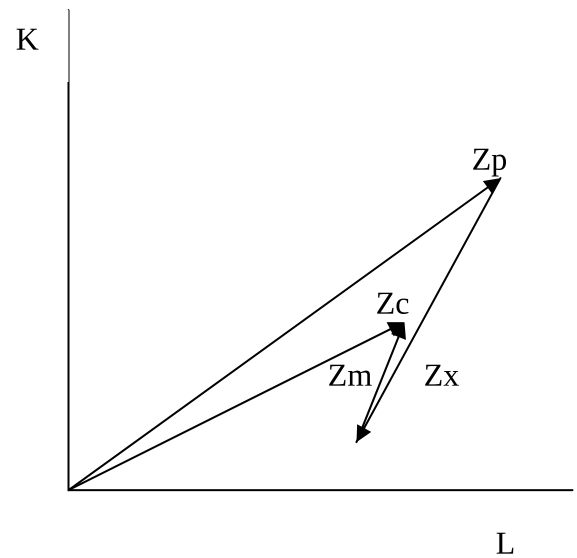
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