Econ 580, Lecture Notes 10

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March 23, 2009

Kremer (1993): Population Growth and Technological Change since 1 Million B.C. (Note: these notes are based on David Romer's *Advanced Economics* texbook, third edition.)

Three equations:

$$Y(t) = T^{\beta}\left[A(t)L(t)\right]^{1-\beta}$$

$$\dot{A}(t) = BL(t)A(t)^{\theta}$$

and

$$Y(t)/L(t) = \bar{y}$$

NOTE: Kremer has $Y = AL^{1-\beta}$. This is equivalent, but changes the appearance of some results below. To see this, let $A_k = A^{1-\beta}$. Then we have $\dot{A}_k(t) = (1-\beta)BL(t)A_k^{(\theta-\beta)/(1-\beta)}$. Thus, Kremer's θ_k equals our $(\theta-\beta)/(1-\beta)$.

Disregard for now the third equation and assume $\theta < 1$. Then a "steady state" entails (using $n = g_L$)

$$\begin{aligned} n + (\theta - 1)g_A &= 0 \\ &\rightarrow g_A = \frac{n}{1-\theta} \end{aligned}$$

We then have

$$\begin{aligned} g_Y &= (1-\beta)(g_A+n) \\ &= (1-\beta)\left(\frac{2-\theta}{1-\theta}\right)n \end{aligned}$$

With $\beta = 0$ we have

$$g_Y - n = \frac{n}{1-\theta}$$

With $\beta > 0$ then

$$\begin{aligned} g_Y - n &= \left[(1-\beta)(2-\theta) - 1 + \theta\right]\frac{n}{1-\theta} \\ &= (1-\beta(2-\theta))\frac{n}{1-\theta} \end{aligned}$$

Now consider third equation, so that $L(t)$ is endogenous - we put aside $\dot{L}/L = n$ (exogenous) and bring in $L(t) = Y(t)/\bar{y}$. Together with the first equation this implies that

$$L(t) = \left(\frac{1}{\bar{y}}\right)^{1/\beta} A(t)^{(1-\beta)/\beta} T$$

Thus,

$$n = \frac{1-\beta}{\beta} g_A$$

We cannot have this and $g_A = \frac{1}{1-\theta} n$ or $n = (1-\theta)$ except if

$$\psi \equiv 1 - (1-\theta)\frac{\beta}{1-\beta} = 0$$

Assume instead that $\psi \neq 0$. Then we cannot have a steady state growth rate. Instead we will have

$$g_A = BL(t)A(t)^{\theta-1}$$

and (from $L(t) = \left(\frac{1}{\bar{\bar{y}}}\right)^{1/\beta} A(t)^{(1-\beta)/\beta} T$)

$$A(t) = aL(t)^{\beta/(1-\beta)}$$

and

$$g_A = \frac{\beta}{1-\beta} n$$

Therefore

$$\begin{aligned}\frac{\beta}{1-\beta}n &= BL(t)\left(aL(t)^{\beta/(1-\beta)}\right)^{\theta-1} \\ &\rightarrow n \sim L(t)^{\psi}\end{aligned}$$

NOTE: Since $\theta_k = (\theta - \beta)/(1-\beta)$ then our ψ can be written as

$$\psi = 1 - (1-\theta)\frac{\beta}{1-\beta} = 1 - (1-\theta_k)\beta$$

Kremer uses notation $\alpha = 1 - \beta$, hence he has that n is proportional to $L^{1-(1-\theta_k)(1-\alpha)}$ (Section II.B).

Data shows that growth rate of population has been increasing together with the population level over the past million years. Thus it would seem that $\psi > 0$.

Moreover, the fact that Eurasia-Africa was more dense than the Americas, Australia and Tasmania around 1500 is consistent with this model.

Land areas of these four regions are 84, 38, 8 and 0.1 square kilometers respectively. Population estimates for the four regions in 1500 imply densities of approximately 4.9 people per square kilometer, 0.4, 0.03 and 0.03 respectively.

Imagine now that L doesn't adjust immediately, but rather that n is a function of y, with $n(\bar{y}) = 0$ for $n'(y) > 0$ for $y \leq \bar{y}$. Kremer assumes that as y increases above $\bar{y}$ eventually $n(y)$ reaches a maximum and starts falling.

It would seem that if $n'(\bar{y})$ is very high then we are close to the simple model above, but Kremer argues that this is not the case, since the simple model is non-robust: once $n'(\bar{y})$ is finite then there is no equilibrium path with $y = \bar{y}$. This is because if $y = \bar{y}$ then $n = 0$ but $g_Y > 0$.

Eventually equilibrium must have growth in y and L, which implies that at some point n starts decreasing. If $n(y)$ reaches a positive constant as y grows large, then the equilibrium asymptotes to the Jones equilibrium. This is inevitable, there is no real Malthusian trap.