Econ 580
Lecture Notes 13

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These lecture notes are about quantifying the aggregate economies of scale arising from the non-rivalry of ideas in quasi-endogenous growth models.

Romer

- A constant growth rate requires a constant rate of technological change. With $Y = F(K, AL)$ and no population growth, then $g = g_A$

- Romer assumes $\dot{A} = AH_I = As_I L$, so $g = s_I L$

- Why is it that $\dot{A}$ increases with A? For Romer, this is because the stock of knowledge increases productivity in R&D (knowledge externalities)

- But, what happens if this benefit is not as strong? One could even think that sustaining a level of $\dot{A}$ is harder when A is higher, since the easy ideas have already been exploited!

Jones

- Jones assumes $\dot{A} = A^{\gamma} s_I L$, $\gamma < 1$, which implies $g = A^{\gamma-1} s_I L$

- When $A \rightarrow \infty$ then $A^{\gamma-1} \rightarrow 0$ and $g \rightarrow 0$. But a growing L can overcome this effect.

- In steady state,

$$g = \left(\frac{1}{1-\gamma}\right) g_L$$

- In levels, this implies that $y = A' L^{1/(1-\gamma)} = A' L^{g/g_L}$

- If $g = 1.5\%$ and $g_L = 4.8\%$ then $g/g_L = 1.5/4.8 = 0.31\,5$, hence

$$y = A' L^{0.315}$$

  Full integration among 100 identical economies implies an increase in y of $100^{0.315} = 4.\,3$.

Kortum

- The following is a simple version of Kortum (1997).

- There is a continuum of intermediate goods $u \in [0, 1]$.

- The final good is produced from these intermediate goods with a CES σ.

- Intermediate goods are produced with labor, with unit-labor cost $x(u)^{\theta}$.

- There is an instantaneous rate of arrival ϕ of ideas per person (exogenous research).

- The stock of ideas is λ, with $\dot{\lambda} = R \equiv \phi L$. In steady state: $\dot{\lambda}/\lambda = g_L$ and $\lambda = R/g_L$.

- Ideas have two elements: the good to which they apply and their "quality."

  - The good to which an idea applies is drawn from a uniform distribution in $u \in [0, 1]$

  - The quality q is drawn from a Pareto distribution with parameter one

- The economy's technology is determined by the best idea available for the production of each good.

- More ideas (higher λ) –> better technology frontier (higher z). Formally, $x \equiv 1/z \sim \exp(\lambda)$

- Letting $p^{1-\sigma} = \int p(u)^{1-\sigma} du$ and assuming $1 + \theta(1-\sigma) > 0$ then (for some constant C)

$$w/p = C\lambda^{\theta}$$

- The growth rate is then

$$g = \theta g_L$$

  – Quasi-endogenous growth

  – Role of θ

- Can do same exercise as above, with same results. Can we put more discipline in this exercise?

A bit more detail

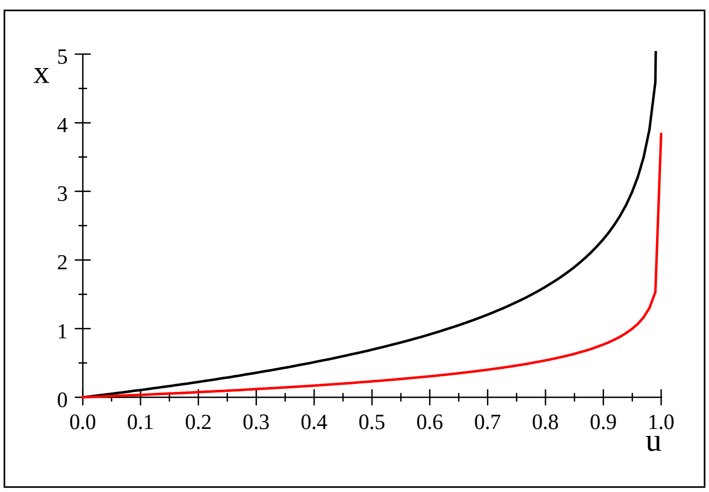
Consider the exponential distribution:

$$F(x) = 1 - e^{-\lambda x}$$

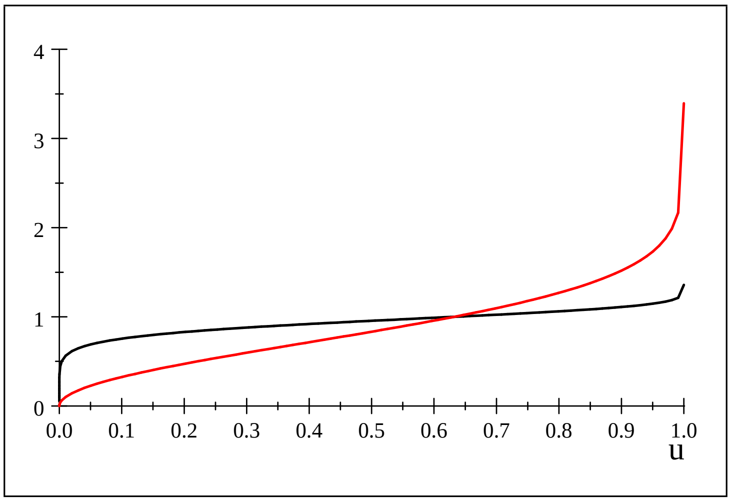
Imagine that $x(u)$ is distributed according to the exponential distribution and order u in such a way that $x(u)$ is increasing. Then we know that $u = F(x)$, hence

$$x(u) = -\frac{1}{\lambda} \ln (1 - u)$$

The respective plot for $\lambda = 1$ and $\lambda = 3$ is



Now consider $x(u)^{\theta}$. Plotting $x(u)^{\theta}$ for $\theta = 1/8$ (black) and $\theta = 1/2$ (red) we have



The expectation of $x(u)^\theta$ when $x(u)$ is distributed according to $F(x) = 1 - e^{-\lambda x}$ is

$$E(x^\theta) = \int_0^\infty x^\theta \lambda e^{-\lambda x} dx$$

Changing variables with $z = \lambda x$ we have $dz = \lambda dx$ and $x^\theta = (z/\lambda)^\theta$, hence

$$\begin{aligned} E(x^\theta) &= \int_0^\infty (z/\lambda)^\theta \, e^{-z} dz \\ &= \lambda^{-\theta} \int_0^\infty z^\theta e^{-z} dz = \lambda^{-\theta} \Gamma(\theta + 1) \end{aligned}$$

where

$$\Gamma(a) \equiv \int_0^\infty z^{a-1} e^{-z} dz$$

is the gamma function. Note: we need to assume that $\theta + 1 > 0$, hence $\theta > -1$. We see that a higher θ implies a bigger impact of an increase in λ on decreasing $E(x^\theta)$. This is true regardless of the behaviour of $\Gamma(\theta - 1)$.

In Kortum we have

$$\begin{aligned} p^{1-\sigma} &= \int_0^1 p(u)^{1-\sigma} du \\ &= \int_0^\infty \left(x^\theta w\right)^{1-\sigma} \lambda e^{-\lambda x} dx \\ &= w^{1-\sigma} \lambda^{-\theta(1-\sigma)} \Gamma(\theta(1-\sigma)+1) \end{aligned}$$

where we have assumed that $\theta(1-\sigma)+1 > 0$. This implies that

$$p = w\lambda^{-\theta} \left[\Gamma(\theta(1-\sigma)+1)\right]^{1/(1-\sigma)}$$

and hence

$$w/p = C\lambda^\theta$$

where

$$C = \left[\Gamma(\theta(1-\sigma)+1)\right]^{1/(\sigma-1)}$$

Quantitative version and implications

- Alvarez and Lucas (2005) enrich the production structure and calibrate the model
  - Intermediate goods are used to produce intermediate goods - labor share β
  - Labor is used to produce the final good with share α
- The only change is that now

$$g = \theta \left(\frac{1 - \alpha}{\beta} \right) g_L$$

  - $1/\beta$ is a multiplier resulting from the circularity in the use of intermediate goods
  - $1 - \alpha$ is the importance of intermediate goods in the production of the final good

- AL parameters: $\alpha = 0.75$ and $\beta = 0.5$, so $(1-\alpha)/\beta = 1/2$

- Using $g_L = 4.8\%$ and $g = 1.5\%$, then need $\theta = 0.63$, hence $g = \theta\left(\frac{1-\alpha}{\beta}\right)g_L = 0.315 g_L$

- In levels, $y \sim L^{0.315} \longrightarrow$ scale effect (Diamond, GGS)
  - If 100 economies integrate perfectly, the income under integration is $100^{0.315} = 4.3$ times higher than under isolation
  - Why? Because there is a higher "stock of ideas," which implies that the best ideas are more productive

- Independent evidence for θ - trade: EK find $\theta = 1/8$; firm-level productivity distribution: $\theta = 1/4$. This presents a puzzle. How to get a higher growth rate?