

# Econ 580, Lecture Notes 2

Andres Rodriguez-Clare

Penn State University

January 22, 2009

Question: where do cross-country differences in  $Y/N$  come from?

Fundamental determinants are “institutions,” or geography, and perhaps even policies

What are the mechanism through which these elements affect labor participation rates ( $L/N$ ) and productivity ( $Y/L$ )?

Most of the variance of  $Y/N$  is due to variance in  $y = Y/L$ , so it makes sense to focus on productivity differences

Two main candidates: capital (broadly speaking) or TFP (efficiency, technology)

Romer talks of "object gaps" vs "idea gaps"

If  $Y = TFP \cdot K^\alpha L^{1-\alpha}$ , then one could in principle just do a variance decomposition analysis for  $y = TFP \cdot (K/L)^\alpha$

But note that some of the variation in  $K/L$  may be due to the variation in  $TFP$

This is because  $r = TFP \cdot \alpha(K/L)^{\alpha-1}$  implies

$$K/L = (\alpha TFP/r)^{1/(1-\alpha)}$$

A better decomposition takes this endogeneity of  $K/L$  into account

Note that  $r = \alpha Y/K$ , so  $k \equiv K/Y = \alpha/r$  is not affected by  $TFP$

We can write  $1 = TFP \cdot (K/Y)^{\alpha}(L/Y)^{1-\alpha}$ , or  $y = TFP^{1/(1-\alpha)}k^{\alpha/(1-\alpha)} \equiv AX$

Note: with  $A$  we write the p.f. as  $Y = K^{\alpha}(AL)^{1-\alpha}$  rather than  $Y = TFP \cdot K^{\alpha}L^{1-\alpha}$

Note: this decomposition doesn't allow  $K$  investment to affect  $A$ ... we will come back to this!

Can differences in  $X$  explain large differences in  $y$  across countries?

The problem is that  $\alpha \approx 1/3$ , so  $\alpha/(1 - \alpha) = 1/2$ . A factor of 10 difference in  $y$  requires a factor of 100 difference in  $K/Y$ . Is this consistent with the data?

Data from the PWT (WDI), at market exchange rates, CR GDP/cap = \$4k, whereas at international dollars it is \$8k, whereas US is the same (normalization)

Other countries: Honduras (\$1k, \$3k), Jamaica (\$3k, \$3.5k), China (\$1k, \$4k)

$K$  constructed by accumulating  $I$ ,

$$K_{t+1} = I_t + (1 - \delta)K_t$$

Initial  $K$  can be constructed by assuming that countries are in steady state in 1960, so that  $K_{1960} = Y_{1960} \cdot k_{1960}$ , with

$$k_{1960} = \frac{I/Y}{\delta + g + n}$$

Here  $I/Y$ ,  $g$  and  $n$  can be calculated for each country according to 1960 – 70, but there are other ways...

How much of the cross-country variation in  $y$  is "accounted for" by the variation in  $k$  and how much by the variation in  $A$ ?

Starting from  $y = Ak$ , then  $\ln y = \ln A + \ln X$ , so

$$\begin{aligned}\frac{\text{var } \ln y}{\text{var } \ln y} &= \frac{\text{cov}(\ln y, \ln A + \ln X)}{\text{var } \ln y} \\ &= \frac{\text{cov}(\ln y, \ln A) + \text{cov}(\ln y, \ln X)}{\text{var } \ln y}\end{aligned}$$

hence

$$1 = \frac{\text{cov}(\ln y, \ln A)}{\text{var } \ln y} + \frac{\text{cov}(\ln y, \ln X)}{\text{var } \ln y} = \hat{\beta}_A + \hat{\beta}_k$$

where  $\hat{\beta}_A$  ( $\hat{\beta}_k$ ) is the coefficient of the OLS regression of  $\ln A$  on  $\ln y$  ( $\ln k$ )

Note: this is simply a variance decomposition when the covariance between  $\ln A$  and  $\ln k$  is shared equally between  $A$  and  $k$ ,

$$\text{var } \ln y = \text{var } \ln A + \text{var } \ln k + 2\text{cov}(\ln A, \ln X)$$

$$1 = \frac{\text{var } \ln A + \text{cov}(\ln A, \ln X)}{\text{var } \ln y} + \frac{\text{var } \ln k + \text{cov}(\ln A, \ln X)}{\text{var } \ln y}$$

$$1 = \frac{\text{cov}(\ln y, \ln A)}{\text{var } \ln y} + \frac{\text{cov}(\ln y, \ln X)}{\text{var } \ln y}$$

**Homework:** go to [klenow.com](http://klenow.com), download the Appendix for KRC ("The Neoclassical Revival...") and do this decomposition. Also graph  $k^{1/2}$  and  $A = y/k^{1/2}$  against  $y$

Perhaps  $k$  is badly measured. Another approach is to think of  $A$  as being common, and then back out  $k$  so that  $y_i = Ak_i^{1/2}$

But then  $k$  would need to vary a lot, generating large differences in  $r_i = \alpha/k_i \dots$  problematic implications for capital flows and convergence (homework: think of these implications)

For the NGT to be a good empirical theory, we need an  $\alpha$  on the order of  $2/3$

In this case  $\alpha/(1 - \alpha) = 2$ , so factor of 10 differences in  $y$  require only factor of around 3 differences in  $k$

But how to get  $\alpha$  up to  $2/3$ ? Human capital!

MRW use  $Y = K^\alpha H^\beta (AL)^{1-\alpha-\beta}$  with  $K$  and  $H$  "produced" in the same way as  $Y$ ,

$$Y = C + I_K + I_H$$

with

$$K_{t+1} = I_{K,t} + (1-\delta)K_t \quad \text{and} \quad H_{t+1} = I_{H,t} + (1-\delta)H_t$$

We can think of this also as

$$\begin{aligned} C &= K_C^\alpha H_C^\beta (AL_C)^{1-\alpha-\beta} \\ I_K &= K_K^\alpha H_K^\beta (AL_K)^{1-\alpha-\beta} \\ I_H &= K_H^\alpha H_H^\beta (AL_H)^{1-\alpha-\beta} \end{aligned}$$

Note also that

$$\begin{aligned} \frac{I_K}{Y} &= \frac{K_K}{K} = \frac{H_K}{H} = \frac{L_K}{L} \\ \frac{I_H}{Y} &= \frac{K_H}{K} = \frac{H_H}{H} = \frac{L_H}{L} \end{aligned}$$

We again have  $y = AX$ , but now

$$X = (K/Y)^{\alpha/(1-\alpha-\beta)} (H/Y)^{\beta/(1-\alpha-\beta)}$$

In steady state, where  $A$  grows at  $g$  and  $Y, K, H$  all grow at  $g + n$  (with  $k$  and  $H/Y$  constant) we have

$$k = \frac{I_{K/Y}}{g + n + \delta} \text{ and } H/Y = \frac{I_{H/Y}}{g + n + \delta}$$

Note treatment of quality (important later on): higher  $A, H/Y$  or  $K/Y$  implies a higher  $H$  given  $L_H/L$

Using the standard measure of  $I_{K/Y}$ , and using the approximation

$$\frac{I_H}{Y} = \frac{L_H}{L} \approx (\text{sec enroll rate}) \cdot \left[ \frac{\text{pop 12-17}}{\text{pop 15-64}} \right]$$

MRW estimate  $\alpha$  and  $\beta$  by running an OLS regression of

$$\ln y_i = a + \frac{\alpha}{1 - \alpha - \beta} \ln(I_{K/Y})_i + \frac{\beta}{1 - \alpha - \beta} \ln(I_{H/Y})_i + \varepsilon_i$$

They find  $\alpha = 0.3$ ,  $\beta = 0.28$  and  $R^2 = 0.78$

Table 1 THE ROLES OF  $A$  AND  $X$  IN 1985 PROSPERITY<sup>a</sup>

Source <sup>a</sup>	$\text{cov}[\ln(Y/L), \ln(Z)]/\text{var} \ln(Y/L)$			
	$Z = \left(\frac{K_Y}{Y}\right)^{\frac{\alpha}{\alpha-\beta}}$	$Z = \left(\frac{H_Y}{Y}\right)^{\frac{\beta}{\alpha-\beta}}$	$Z = X$	$Z = A$
MRW0	29	49	78	22
MRW1	.27	49	76	.24
MRW2	31	47	78	.22
MRW3	.29	11	40	60
MRW4	29	04	33	67

<sup>a</sup>MRW0 from MRW (uses their data appendix) MRW1 MRW0 but with  $K_Y, Y$  instead of  $K/Y$  MRW2 MRW1 but with  $L =$  worker instead of working-age population, 14 countries in/out MRW3 MRW2 but with all enrollment rather than just secondary enrollment MRW4 MRW3 but with  $(k, H, L)$  shares of (0.1, 0.4, 0.5), not (0.20, 0.28, 0.42), in  $H$  production

What about correlation between investment rates and TFP?

Not clear how to think of  $\beta$

Better to use independent measures of  $\alpha$  and  $\beta$ ...  $\alpha = 0.3$  is fine, not clear about  $\beta$

There is a better way to incorporate human capital (below), but use  $\alpha = 0.3, \beta = 0.28$  for now

MRW1 - measured output is not  $C + I_K + I_H$ ... part of  $I_H$  goes unmeasured (value of student's time)

Assume that in fact measured  $Y$  is simply  $C + I_K$ . Then what matters for explaining  $Y/L$  is not  $K/Y$  and  $H/Y$  but rather  $K_Y/Y$  and  $H_Y/Y$ , where  $K_Y \equiv K_C + K_K$  and  $H_Y = H_C + H_K$ .

In KRC we measure  $K_Y = (1 - L_H/L)K$  and  $H_Y = (1 - L_H/L)H$

MRW2 - better data

MRW3 - overall enrollment rates,

$$\frac{I_H}{Y} = \frac{L_H}{L} = (\text{enroll rate}) \cdot \left[ \frac{\text{pop 7-19}}{\text{pop 15-64}} \right]$$

MRW4 - human capital accumulation is less capital intensive than the production of  $C$  and  $I_K$ ,

$$I_H = K_H^{1-\phi-\lambda} H_H^\phi (AL_H)^\lambda$$

Evidence suggests that the physical capital share is 0.1 and the human capital share is 0.4 (i.e.,  $\lambda = 0.5$  and  $\phi = 0.4$ ).

Two opposite effects on  $H_Y/Y$ : (1) higher  $L_H/L$  leads to more  $H_Y/Y$  than in MRW, but (2) higher  $K_Y/Y$  leads to less  $H_Y/Y$ . Empirically, this implies that  $H_Y/Y$  varies less

In sum, from 78% vs 22% decomposition in MRW1, to 33% to 67% decomposition in MRW4

Problem: if human capital investment equals schooling, then we cannot model human capital as some disembodied capital

Think of schooling as an investment of the first  $s$  years of finite lives (for now exogenous)

Differences in wages associated with differences in  $s$  tell us about the returns to schooling... no need for  $\beta$ : use  $Y = K^\alpha (AH)^{1-\alpha}$  with  $H = \int_{\Omega} h(s) dF(s)$

It is reasonable to assume that  $h(s) = Bw(s)$  so that  $h$  is proportional to  $w$ ... microeconomic research (Mincer regression) tells us that  $w(s) \sim e^{\gamma s} \rightarrow$  each additional year of schooling increases an individual's wage by approximately the same percentage amount (Bils and Klenow, 2000)

We then have  $y = AX$ , with  $X = k^\alpha / (1-\alpha) h$

Using  $\phi = 0.095$  (BK), KRC find a decomposition of 34% vs 64% (BK4)

Note: KRC actually use  $Y = K^\alpha H^\beta (AL)^{1-\alpha-\beta}$  with  $\alpha = 0.3$  and  $\beta = 0.28$ ... results don't change significantly (homework: check!)

Similar results in Hall and Jones (1999)

What about quality differences in schooling across countries?

Using

$$h(s) = (K_H/L_H)^{1-\phi-\lambda} h_T^\phi \left( A e^{(\gamma/\lambda)s} \right)^\lambda$$

then

$$quality = (K_H/L_H)^{1-\phi-\lambda} h_T^\phi A^\lambda$$

With  $\lambda = 0.5$  and  $\phi = 0.4$ , KRC find a decomposition of 56% vs 43% (BK2)

But this implies large differences in quality of schooling, with a "statistical elasticity" of quality w.r.t.  $y$  of 0.95%

Borjas (1987) uses wage data for immigrants in U.S. from different countries and finds an elasticity closer to 12%

A closer approximation to this world is to assume  $\lambda = 1$  and  $\phi = 0$  as in BK4

Relation to Hendricks (2002)