Econ 580, Lecture Notes 3

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According to KRC, 23% of cross-country income differences come from variation in K/Y.

Question: what explains the cross-country variation in K/Y?

Paper: Hsieh and Klenow (2007) (related: Caselli and Feyrer, 2007)

Figure 1: Investment Rates at International Prices

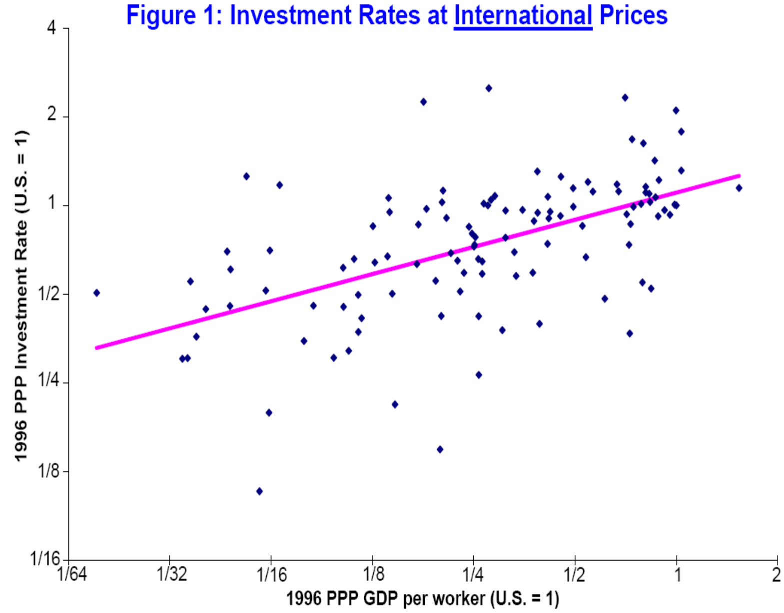
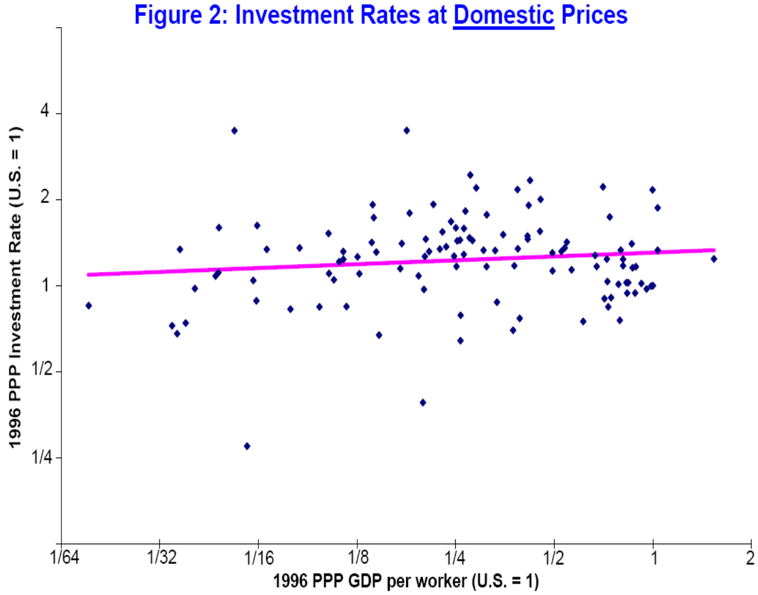


Figure 2: Investment Rates at Domestic Prices



Model with tradable investment and nontraded consumption

Instead of country subscripts, think of small country and international prices with *

Production of C and I according to

$$\begin{aligned} C &= A_C K_C^{\alpha} L_C^{1-\alpha} \\ I &= A_I K_I^{\alpha} L_I^{1-\alpha} \end{aligned}$$

A_C and A_I are exogenous and grow at a common rate g_A

It is easier to view this by thinking that capital and labor produce an input X with technology $X = K^{\alpha} L^{1-\alpha}$

This implies a unit cost of X of

$$c_x = \left(\frac{R}{\alpha}\right)^{\alpha} \left(\frac{w}{1-\alpha}\right)^{1-\alpha}$$

This input is then used with productivity A_C and A_I to produce C and I, respectively.

Tax τ_I levied on sales of investment goods at retail level (not a tariff!), consumer price is

$$P_I = P_I^*(1 + \tau_I)$$

Local producers of investment goods get (producer price)

$$\frac{P_I}{1 + \tau_I} = P_I^*$$

Note: producer price is independent of the tax, which falls completely on consumers. Why?

Note: implicitly we have set the numeraire in the small economy to be the same as the numeraire in the international economy (which defines P_I^*). This implies that the exchange rate is one. If we choose some other numeraire, then there will be an exchange rate $e \neq 1$, but all results hold (check!)

Zero-profit condition for investment-good producers entails

$$P_I^* = \frac{P_I}{1 + \tau_I} = \frac{c_x}{A_I}$$

Similarly,

$$P_C = \frac{c_x}{A_C}$$

The ratio of producer prices is the same as in a closed economy,

$$\frac{P_I^*}{P_C} = \frac{A_C}{A_I}$$

but the ratio of consumer prices is affected by the tax,

$$\frac{P_I}{P_C} = \frac{A_C(1 + \tau_I)}{A_I}$$

Consumer/workers own capital, rent labor and capital to firms, pay retail taxes when they buy investment goods, and pay capital taxes. They buy investment goods to accumulate capital according to

$$K(t+1) = (1-\delta)K(t) + I(t)$$

Firms rent capital up to the point where

$$R = \alpha P_I^* A_I \left(\frac{K_I}{L_I}\right)^{\alpha-1} = \alpha P_C A_C \left(\frac{K_C}{L_C}\right)^{\alpha-1}$$

so clearly

$$K_I/L_I = K_C/L_C = K/L$$

Note: this is the same as simply saying that K and L are used to produce the input which is then sold to producers of investment and consumption goods.

The condition $P_I^* A_I = c_x$ implies

$$P_I^* A_I = \left(\frac{R}{\alpha}\right)^{\alpha} \left(\frac{w}{1-\alpha}\right)^{1-\alpha}$$

Together with

$$R = \alpha P_I^* A_I \left(\frac{K}{L}\right)^{\alpha-1}$$

we can now determine w and R as functions of P_I^* and K.

Preferences (no disutility of labor, inelastic supply)

$$\sum \beta^t \frac{C(t)^{1-1/\sigma}}{1-1/\sigma}$$

If there were no taxes, the budget constraint would be

$$P_C C + P_I^* I = w + RK$$

But with investment and capital-income taxes we have

$$P_C C + \frac{P_I^* I}{1+\tau_I} = w + \left[R - \tau_K \left(R - \delta \frac{P_I^* I}{1+\tau_I}\right)\right] K + T$$

A final condition relates the rental rate, R, with the price of investment, P_I: a unit of investment today costs $P_I(t)$, next period have an extra unit of capital, which yields

$$R(t+1)-\tau_K(R(t+1)-\delta P_I(t+1))+(1-\delta)P_I(t+1)$$

This must be equal to what we could have obtained from investing $P_I(t)$, $(1+r(t+1))P_I(t)$, so

$$\begin{aligned} & R(t+1)-\tau_K(R(t+1)-\delta P_I(t+1)) \\ & +(1-\delta)P_I(t+1)=(1+r(t+1))P_I(t) \end{aligned}$$

But since $P_I(t)$ is constant, then we have

$$R=\frac{r+\delta(1-\tau_K)}{1-\tau_K}P_I \tag{1}$$

The rental R is increasing in P_I, r, and the capital-income tax τ_K

What is the interest rate r in steady state?

The Euler equation is

$$\frac{\beta u'(C(t+1))}{u'(C(t))} = \frac{1}{1+r}$$

Since C grows at rate g, then we have

$$r = \frac{(1+g)^{1/\sigma}}{\beta} - 1 \qquad (2)$$

Note that this is the same in all countries if g, σ, and β is the same. It doesn't depend on taxes... why?

Definitions:

$$
\begin{aligned}
\text{Domestic Price } GDP &= P_C C + P_I I \\
\text{International Price } GDP &= P_C^* C + P_I^* I \\
\text{Domestic Price Inv. Rate } i_{dom} &= \frac{P_I I}{P_C C + P_I I} \\
\text{International Price Inv. Rate } i &= \frac{P_I^* I}{P_C^* C + P_I^* I}
\end{aligned}
$$

Note that we can then get that

$$i_{dom} = i \cdot \frac{P_I / P_I^*}{(P_C C + P_I I)/(P_C^* C + P_I^* I)} = i \cdot \frac{P_I / P_I^*}{E}$$

where E is the PPP exchange rate. Also,

$$i_{dom} = i \left[(1 - i_{dom}) \frac{P_I / P_C}{P_I^* / P_C^*} + i_{dom} \right]$$

To characterize the equilibrium, first solve for the share of labor (and capital) devoted to investment goods production, L_I/L.

To do this, note that $RK_C = \alpha P_C C$ and $RK_I = \alpha P_I^* I$, so $RK = \alpha \widetilde{Y}$, where $\widetilde{Y}$ is GDP at local producer prices.

This implies that $K/\widetilde{Y} = \alpha/R$, and hence that $I/\widetilde{Y} = \alpha(\delta + g)/R$

In standard one sector model (i.e., $C + I = AK^\alpha L^{1-\alpha}$) we have $L_I/L = I/Y = I/\widetilde{Y} = \alpha(\delta + g)/R$, but now we have

$$\begin{aligned} \frac{I}{\widetilde{Y}} &= \frac{I}{P_C C + P_I^* I} = \frac{A_I X_I}{P_C A_C X_C + P_I^* A_I X_I} \\ &= \frac{X_I / P_I^*}{\frac{P_C A_C}{P_I^* A_I} X_C + X_I} = \frac{X_I / P_I^*}{X_C + X_I} = \frac{L_I}{L} \frac{1}{P_I^*} \end{aligned}$$

Hence,

$$\frac{1}{P_I^*} \frac{L_I}{L} = \frac{\alpha(\delta + g)}{R}$$

Using the equilibrium conditions (1) and (2) above, then we get equation (9) in HK,

$$\frac{L_I}{L} = \frac{(\delta + g)\alpha\,(1 - \tau_K)}{(1 + \tau_I)\left[(1 + g)^{1/\sigma}/\beta - 1 + \delta(1 - \tau_K)\right]} \qquad (3)$$

This implies that L_I/L is decreasing in τ_I and τ_K

Simple algebra shows that (this is equation (10) in HK)

$$i_{dom} = \frac{(1 + \tau_I)L_I/L}{1 + \tau_I L_I/L} \qquad (4)$$

and

$$i = i_{dom}\left[(1 - i_{dom})\frac{A_C(1 + \tau_I)}{A_I}\frac{P_C^*}{P_I^*} + i_{dom}\right]^{-1}$$

Implications

Both i_{dom} and i are strictly decreasing in τ_K. This is simple because τ_K does not affect relative prices. It increases R and hence decreases K/L, which entails lower i_{dom} and i.

A higher τ_I also decreases i, similar logic: it increases R and hence must decrease K/L and i.

But a higher τ_I also increases P_I/P_C, since

$$\frac{P_I}{P_C} = \frac{A_C(1+\tau_I)}{A_I}$$

The effect on i dominates, so i_{dom} ends up being lower. Mechanically, we know that $(1+\tau_I)L_I/L$ doesn't change, so $\tau_I L_I/L$ must increase when τ_I increases (and L_I/L falls).

NOTE: the effect on i is stronger than on i_{dom}

i_{dom} does not depend on sectoral TFPs (prices and quantities exactly offset each other), but low TFP in the investment sector relative to TFP in consumption decreases i (no offsetting price effect)

Table 1

PPP Investment Rates vs. Investment Rates at Domestic Prices

Independent Variable = PPP GDP per worker

Dependent Variable	PPP Investment Rates			Investment Rates at Domestic Prices		
	1980	1985	1996	1980	1985	1996
Fixed Investment	**.30**	**.52**	**.31**	**.09**	**.11**	.06
	(.06)	(.05)	(.04)	(.04)	(.03)	(.04)
	R^2 = .37	R^2 = .60	R^2 = .32	R^2 = .10	R^2 = .15	R^2 = .02
Producer Durables	**.28**	**.58**	**.43**	.04	**.10**	.04
	(.07)	(.06)	(.08)	(.05)	(.04)	(.06)
	R^2 = .24	R^2 = .60	R^2 = .28	R^2 = .02	R^2 = .08	R^2 = .01
Structures	**.34**	**.49**	**.36**	**.13**	**.13**	**.26**
	(.07)	(.06)	(.09)	(.04)	(.04)	(.09)
	R^2 = .32	R^2 = .51	R^2 = .16	R^2 = .14	R^2 = .18	R^2 = .13
# of benchmark countries	61	64	114	61	64	114

Notes: All variables are in logs. Each entry is a coefficient from a single regression. Robust standard errors are in parentheses. Bold coefficients are significant at the 5% level. Fixed Investment includes producer durables and structures, and excludes inventory investment. Producer durables include machinery and equipment (including electrical nonelectrical machinery) and vehicles.

Table 2

The Price of Investment Goods

Independent Variable = PPP GDP per worker

Dependent Variable	At *Official* Exchange Rates			At *Black Market* Exchange Rates		
	1980	**1985**	**1996**	**1980**	**1985**	**1996**
Fixed Investment	.024	−.038	**.183**	**.190**	.096	**.245**
	(.049)	(.048)	(.047)	(.053)	(.050)	(.043)
	R^2 = .00	R^2 = .01	R^2 = .13	R^2 = .14	R^2 = .03	R^2 = .19
Producer Durables	−.006	**−.142**	.052	**.055**	**−.085**	**.113**
	(.034)	(.035)	(.032)	(.039)	(.035)	(.030)
	R^2 = .00	R^2 = .14	R^2 = .02	R^2 = .03	R^2 = .05	R^2=0.08
Structures	.016	−.029	**.339**	**.181**	.105	**.401**
	(.068)	(.061)	(.060)	(.067)	(.066)	(.055)
	R^2 = .00	R^2 = .00	R^2 = .18	R^2 = .09	R^2 = .03	R^2 = .23
# of benchmark countries	61	64	114	61	64	114

Notes: All variables are in logs. Each entry is a coefficient from a single regression. Robust standard errors are in parentheses. Bold coefficients are significant at the 5% level. The dependent variable is the log investment price expressed in dollars (converted from national currencies at official or black market exchange rates).

Figure 4: 1996 Price of Producer Durables

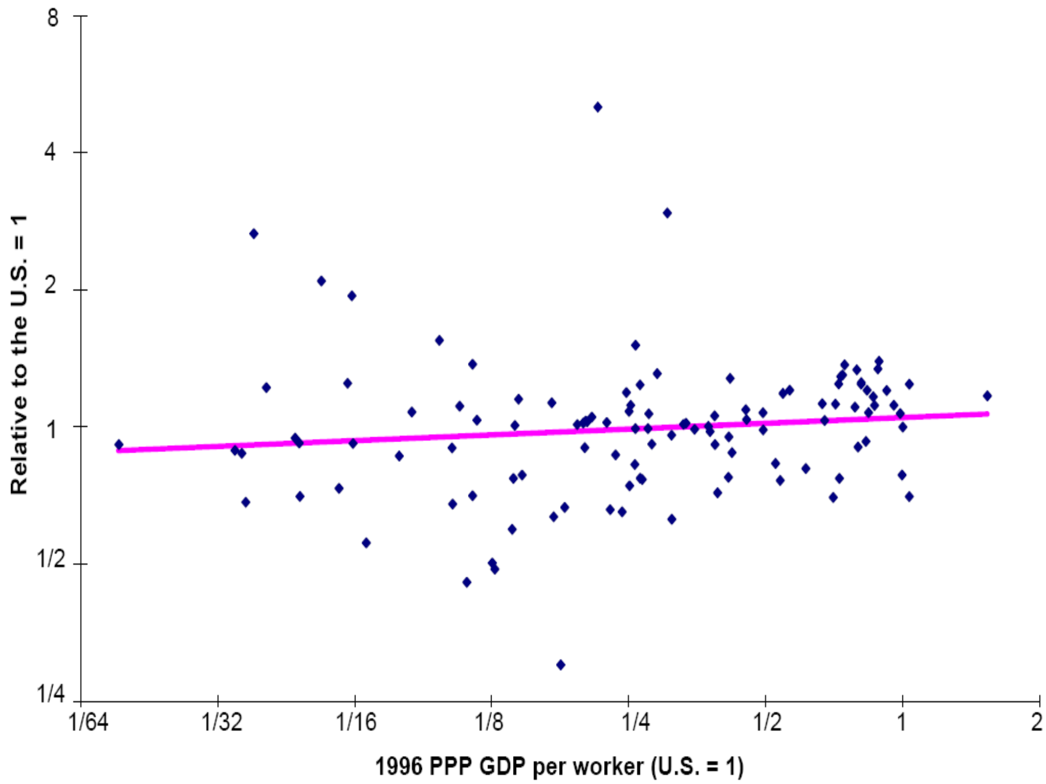


Figure 5: 1996 Price of Consumption

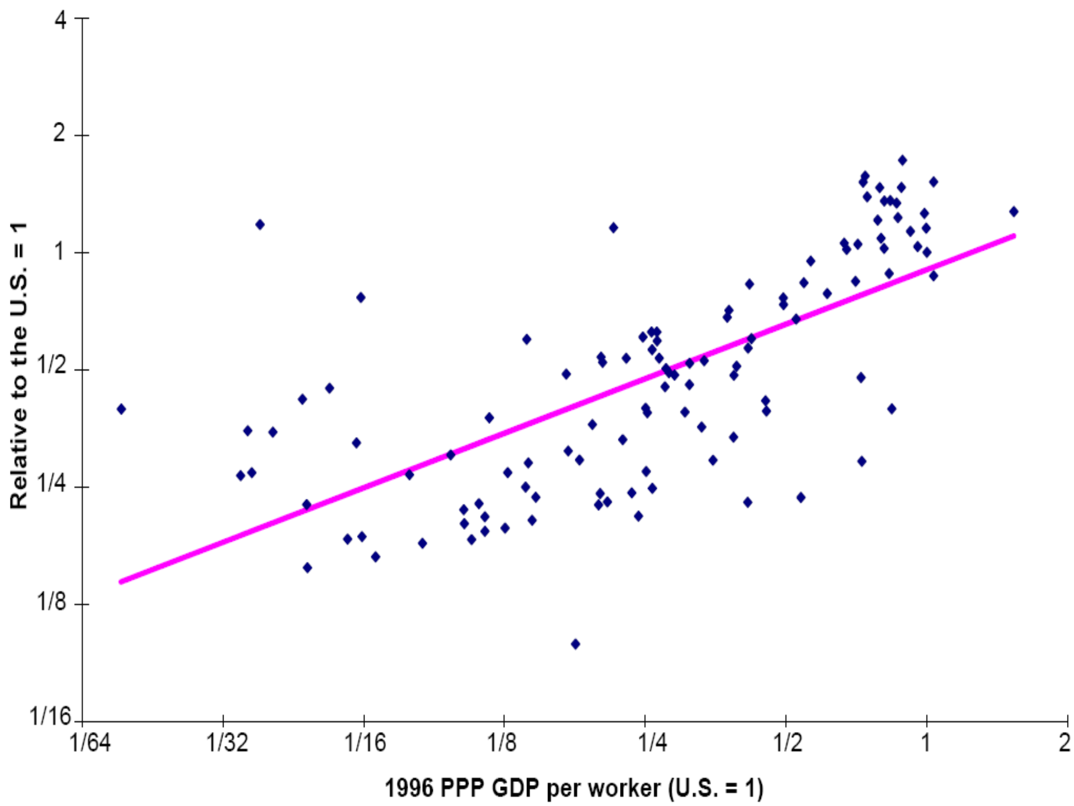


Table 3

The Price of Consumption

Independent Variable = PPP GDP per worker

Dependent Variable ↓	At ***Official*** Exchange Rates			At ***Black Market*** Exchange Rates		
	1980	**1985**	**1996**	**1980**	**1985**	**1996**
All Consumption	**.221**	**.286**	**.446**	**.387**	**.420**	**.507**
	(.053)	(.049)	(.057)	(.048)	(.039)	(.056)
	R^2 = .25	R^2 = .41	R^2 = .43	R^2 = .43	R^2 = .46	R^2 = .46
Consumption Services	**.377**	**.415**	**.660**	**.542**	**.549**	**.721**
	(.064)	(.050)	(.070)	(.062)	(.050)	(.073)
	R^2 = .38	R^2 = .51	R^2 = .48	R^2 = .49	R^2 = .53	R^2=.52
Consumption Goods	**.141**	**.223**	**.310**	**.307**	**.357**	**.372**
	(.047)	(.049)	(.048)	(.045)	(.034)	(.044)
	R^2 = .15	R^2 = .33	R^2 = .35	R^2 = .37	R^2 = .41	R^2=.42
# of benchmark countries	61	64	114	61	64	114

Notes: All variables are in logs. The dependent variable is a log consumption price expressed in dollars (converted from national currencies at official or black market exchange rates). Nontradables are services, tradables are goods.

Table 6

Decomposing PPP Investment Rates

Independent Variable = PPP GDP per worker

Dependent Variable	1980	1985	1996
Actual PPP investment rate	**.298** (.051)	**.520** (.054)	**.311** (.043)
PPP investment rate with only …			
… γ_C varying	**-.033** (.004)	**-.038** (.003)	**-.050** (.003)
… $(1-\tau_K)$ varying	**.079** (.033)	**.097** (.032)	.048 (.032)
…$(1+\tau_I)$ varying	.025 (.013)	**.079** (.016)	.019 (.011)
… A_T/A_S and A_T/A_N varying	**.252** (.033)	**.283** (.030)	**.276** (.035)
residual	-.025	.099	.018
# of benchmark countries	61	64	114

Notes: All variables are in logs. Each entry is a coefficient from a single regression. Standard errors are in parentheses. Bold coefficients are significant at the 5% level. Investment refers to fixed investment. See equations (31) and (32) for how each variable listed affects the PPP investment rate. We set variables to their U.S. values except for in the term listed. In constructing this Table, we used prices converted into dollars at *official* exchange rates.

Table 7

Productivity Levels and Income Differences

(Entries are elasticities with respect to PPP Y/L)

	1980	1985	1996
A_T	**.739** (.049)	**.714** (.047)	**.962** (.047)
A_N	**.723** (.039)	**.743** (.040)	**.622** (.048)
A_S	**.363** (.030)	**.299** (.025)	**.303** (.029)
TFP as a result of *A* variation	**.700** (.031)	**.606** (.031)	**.624** (.034)
K/Y as a result of *A* variation	**.126** (.016)	**.141** (.015)	**.138** (.017)
# of countries	61	64	114

Notes: All variables are in logs. Each entry is a coefficient from a single regression. Standard errors are in parentheses. Bold coefficients are significant at the 5% level. For the last two rows we calculated TFP and K/Y using country TFPs in each sector but setting all other variables (spending shares on durables, tax rates) to U.S. levels. In constructing this Table, we used prices converted into dollars at *official* exchange rates.