Econ 580, Lecture Notes 4

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January 27, 2009

BK4 in KRC assume that s is exogenous and there are no quality differences. In BK4, s variation explains 11% of variation in income levels... but what drives s variation? We also want to allow for endogenous differences in schooling quality. Paper: Erosa, Koreshkova and Restuccia, 2007.

In Bils and Klenow and KRC, it is assumed that $w = h = Be^{\gamma s}$ is a structural relationship, so data (i.e., Mincer regression) pins down the structural parameter γ

This is done with the average Mincer coefficients among 50+ countries in Bils and Klenow. Note that this implies that "quality" of education is the same across countries ...although B can vary (as in KRC)

It is also assumed that s is exogenous. But why does s differ across countries?

One possibility: differences in life expectancy and discount rates (as in homework). But then γ would be higher in poorer countries

Want a model where quality of education is a function of expenditures in schooling, which will depend on A and K, and where s is endogenous. We can then use data on Y, K and s to infer H.

Note: another important paper here is Manuelli and Sheshadri (2005, Human Capital and the Wealth of Nations) and more recently Córdoba and Ripoll (who presents?)

Basic idea: consider individuals that live for two periods, young and adult

When young they go to school for s years, then they are idle until they become adults and work

Education for s years entails hiring teachers at cost $w\bar{l}s$. This implies using $\bar{l}$ units of human capital per "year" of schooling

With education spending on "materials" equal to e then human capital when adult for an individual of ability z is

$$h = zf(s, e)$$

where $f(s, e)$ is increasing and concave

Production when adult is simply wh

An agent of ability z will choose s and e to maximize

$$\max_{s,e}\left\{\beta wzf(s,e) - e - w\bar{l}s\right\}$$

First order conditions are

$$\begin{aligned}\beta wzf_s &= w\bar{l} \\ \beta wzf_e &= 1\end{aligned}$$

Note that the wage has no DIRECT effect on s because marginal cost increases just as much as marginal benefit

An increase in z or w will increase both s and e

Key intuition: higher wage increases benefits of expenditures in education relative to cost... higher TFP decreases cost of schooling expenditures (structures, books, etc.) relative to wages... true in the data?

Now assume that

$$f(s,e) = (s^{\eta} e^{1-\eta})^{\xi}$$

Note that

$$\frac{\partial \ln f}{\partial s} = \xi \eta / s$$

which is decreasing, whereas the Mincer regression suggests that $d \ln h / ds$ is constant or even increasing

But there is no contradiction... why?

Using $f(s,e) = (s^{\eta} e^{1-\eta})^{\xi}$ and the F.O.C.'s it is easy to show that

$$E_{sw} \equiv \frac{\partial \ln s}{\partial \ln w} = \frac{(1-\eta)\xi}{1-\xi}$$

Note that if $\eta = 1$ then $E_{sw} = 0$... why?

We also get

$$E_{ew} = \frac{1}{1-(1-\eta)\xi} \left(1 + \eta\xi \frac{(1-\eta)\xi}{1-\xi} \right)$$

and hence

$$E_{hw} = \xi\eta E_{sw} + \xi(1-\eta) E_{ew} = \frac{(1-\eta)\xi}{1-\xi}$$

A positive E_{hw} requires $\eta < 1$. Also note that E_{hw} increases with the elasticity parameter ξ

Let's now embed this into a model with

$$Y = AK^{\alpha}H^{1-\alpha}$$

Assume that $Y = C + X + E$, with

$$K' = (1 - \delta)K + X$$

There is no growth and the discount rate is β, so in steady state $r = 1/\beta - 1$

Authors use notation ρ instead of r

Recall that in equilibrium we will have the rental rate equal to $(\rho + \delta)$

Then in equilibrium we have

$$(\rho + \delta) = A\alpha(K/H)^{\alpha-1}$$

This implies that

$$\frac{K}{H} = \left(\frac{\alpha A}{\rho + \delta}\right)^{1/(1-\alpha)}$$

so

$$K = c_k H A^{1/(1-\alpha)}$$

which implies that

$$E_{KA} = \frac{1}{1-\alpha} + E_{HA}$$

Note amplification through H

We now have that

$$E_{YA} = 1 + \alpha E_{KA} + (1-\alpha)E_{HA}$$

so

$$E_{YA} = \frac{1}{1-\alpha} + E_{HA}$$

But

$$w = (1-\alpha)A(K/H)^{\alpha} = (1-\alpha)A^{1/(1-\alpha)}c_k^{\alpha/(1-\alpha)}$$

and we know from above that (using $E_{Hw} = E_{hw}$)

$$E_{HA} = E_{hw}E_{wA} = \frac{(1-\eta)\xi}{1-\xi} \cdot \frac{1}{1-\alpha}$$

Putting it all together, we finally have

$$E_{YA} = \frac{1}{1-\alpha}\left(\frac{1-\eta\xi}{1-\xi}\right)$$

Homework (for Thursday Jan 29): assume that

$$K' = (1-\delta)K + A_k X$$

Note that here A_k plays the role of A_I/A_C in Hsieh and Klenow (2007). Calculate E_{YA_k}.

Quantitative implications of

$$E_{YA} = \frac{1}{1-\alpha}\left(\frac{1-\eta\xi}{1-\xi}\right)$$

Imagine that $\alpha = 1/3$ and $\xi = 0.8$ and $\eta = 0.75$. Then $E_{YA} = 3$

This implies a multiplier of TFP ratios Δ of Δ^3, whereas with $E_{HA} = 0$ we have $\Delta^{1.5}$

A TFP difference of 3 implies income differences of a factor of $3^3 = 27$!

Standard amplification is only $3^{1.5} = 5$... human capital represents an important source of amplification