Econ 580, Lecture Notes 5

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We need two things to explain large income differences: TFP differences and strong amplification

ERK offer amplification through schooling quality. Jones notes the role of intermediate goods in both amplification and TFP differences

Paper: Jones, 2008 (Intermediate Goods, Weak Links...)

Amplification through capital

Imagine that

$$
\begin{aligned}
Y_t &= AK_t^\alpha = C_t + I_t \\
K_{t+1} &= I_t = sY_t
\end{aligned}
$$

Assume we start with some exogenous K_0. Then

$$
\begin{aligned}
Y_0 &= AK_0^\alpha \\
Y_1 &= A\left(sY_0\right)^\alpha = A\left(sAK_0^\alpha\right)^\alpha = A^{1+\alpha}s^\alpha K_0^{\alpha^2} \\
Y_2 &= A\left(sY_1\right)^\alpha = A^{1+\alpha+\alpha^2}s^{\alpha+\alpha^2}K_0^{\alpha^3} \\
Y_\infty &= A^{1/(1-\alpha)}s^{\alpha/(1-\alpha)}
\end{aligned}
$$

The role of intermediate goods

Imagine that

$$Y_G = A\left(K^{\alpha}L^{1-\alpha}\right)^{1-\sigma}X^{\sigma}$$

where

$$\begin{aligned} Y_G &= C + I + X \\ K' &= (1-\delta)K + I \end{aligned}$$

Notice loop: X used in production of X. This is a modeling trick... could do dynamic, with X as a capital good that depreciates completely in one period, same result in steady state.

The price of X is 1, so

$$A\sigma\left(K^{\alpha}L^{1-\alpha}\right)^{1-\sigma}X^{\sigma-1}=1$$

hence

$$X=\sigma Y_G$$

so

$$Y_G=A'K^{\alpha}L^{1-\alpha}$$

where

$$A'=(\sigma^{\sigma}A)^{1/(1-\sigma)}$$

But GDP is $Y=C=(1-\sigma)Y_G$, so GDP per capita is

$$y=(1-\sigma)\left(A'\right)^{1/(1-\alpha)}(K/Y)^{\alpha/(1-\alpha)}$$

where K/Y depends on preferences, δ and g_A

Then we have

$$y \sim A^{1/(1-\sigma)(1-\alpha)} (K/Y)^{\alpha/(1-\alpha)}$$

so now

$$E_{yA} = \frac{1}{(1-\sigma)(1-\alpha)} = \frac{1}{1-[\alpha+\sigma(1-\alpha)]}$$

Data suggests that $\sigma = 1/2$. Hence $\alpha = 1/3$ implies that $E_{yA} = 3$ rather than 3/2.

Introducing intermediate goods doubles the amplification effect, just like increasing α from 1/3 to 2/3

Consider the effect of a tax τ on sales of X. Using output Y as the numeraire, then in equilibrium we must have $(1-\tau)p = 1$, so $p = 1/(1-\tau)$.

This implies that now

$$A\sigma \left(K^{\alpha}L^{1-\alpha}\right)^{1-\sigma} X^{\sigma-1} = 1/(1-\tau)$$

hence $X = \sigma(1-\tau)Y$

But then

$$\begin{aligned} Y_G &= A\left(K^{\alpha}L^{1-\alpha}\right)^{1-\sigma}(\sigma(1-\tau)Y)^{\sigma} \\ &\rightarrow Y_G = (A\sigma^{\sigma}(1-\tau)^{\sigma})^{1/(1-\sigma)} K^{\alpha}L^{1-\alpha} \end{aligned}$$

hence

$$Y = (1-\sigma(1-\tau))(A\sigma^{\sigma}(1-\tau)^{\sigma})^{1/(1-\sigma)} K^{\alpha}L^{1-\alpha}$$

It is easy to see that Y is maximized for $\tau = 0$.

Weak links and superstars

Imagine that

$$Y = \left(\int_0^1 x(s)^\beta ds \right)^{1/\beta}$$

The elasticity of substitution is

$$\varepsilon = \frac{1}{1-\beta}$$

If $\beta = 0$ then $\varepsilon = 1$, Cobb Douglas

If $\beta \in (0, 1)$ then $\varepsilon > 1$, substitutes

* As $\beta \rightarrow 1$ then $\varepsilon \rightarrow \infty$, perfect substitutes

If $\beta < 0$ then $\varepsilon < 1$

* As $\beta \rightarrow -\infty$ then $\varepsilon \rightarrow 0$, perfect complements (Leontief)

Assume that

$$x(s) = A(s)L(s)$$

Resource constraint

$$\int_0^1 L(s)ds = L$$

What is the allocation of labor? Use CES results.

Since $p(s) = w/A(s)$, then the price index is

$$\begin{aligned} P^{1-\varepsilon} &= \int_0^1 p(s)^{1-\varepsilon} ds \\ w/P &= \left(\int_0^1 A(s)^{\varepsilon-1} ds \right)^{1/(\varepsilon-1)} \\ &= \left(\int_0^1 A(s)^{\eta} ds \right)^{1/\eta} \end{aligned}$$

This is a power mean of $A(s)$ with curvature parameter η

What is $L(s)$? Letting $w = 1$, then $Y = L$, and spending on s is

$$E(s) = \frac{p(s)^{1-\varepsilon}}{\int_0^1 p(s)^{1-\varepsilon} ds} L = \frac{A(s)^{\varepsilon-1}}{P^{1-\varepsilon}} L$$

But $p(s) = w/A(s)$ implies

$$E(s) = p(s)q(s) = p(s)A(s)L(s) = wL(s) = L(s)$$

hence

$$L(s)/L = P^{\varepsilon-1} A(s)^{\varepsilon-1}$$

If $\varepsilon = 1$ then $L(s)$ same for all s

If $\varepsilon > 1$ (substitutes) then $L(s)$ increases in $A(s)$ –> *superstars*

If $\varepsilon < 1$ (complements) then $L(s)$ decreases in $A(s)$ –> *weak links*

Consider the symmetric allocation $L(s) = L$, perhaps due to distortions

Then

$$\begin{aligned} Y &= \left(\int_0^1 x(s)^\beta ds \right)^{1/\beta} \\ &\implies Y/L = w/P = \left(\int_0^1 A(s)^\beta ds \right)^{1/\beta} \end{aligned}$$

So we go from curvature parameter $\varepsilon - 1 = \beta/(1-\beta)$ (equilibrium) to β (S.A.)

Note that $\beta < \beta/(1-\beta)$, w/P is higher in equlibrium than in the S.A.

If $\beta \to -\infty$ so $w/P \to \min\{A(s)\}$ in the S.A. whereas in equilibrium w/P is the harmonic mean of $\{A(s)\}$