Econ 580, Lecture Notes 6

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The standard development accounting exercise uses $Y = K^{\alpha}(AH)^{1-\alpha}$ and finds large differences in A.

One possibility is that poor countries are abundant in unskilled labor, and that the "world technology frontier" is skewed towards skilled labor. Poor countries would then choose points that reflect their endowments, but this would be captured as a low A.

This is one way to understand the paper by Caselli and Coleman titled "The World Technology Frontier."

The standard approach with $Y = K^{\alpha}(AH)^{1-\alpha}$ assumes that labor of different skill levels are perfect substitutes, so they can be aggregated into H by simply adding the $h's$ - $H = \int h dF(h)$.

Caselli and Coleman (CC) argue that in fact there are two qualitatively different groups: unskilled and skilled. These different workers perform entirely different tasks, so it is impossible to replace a skilled worker with a number of unskilled workers,

$$Y = K^{\alpha}((A_u H_u)^{\sigma} + (A_s H_s)^{\sigma})^{(1-\alpha)/\sigma}$$

Note: I changed the notation from L_u and L_s to H_u and H_s.

If $\sigma \rightarrow 1$ then have

$$Y = K^{\alpha}(A_u H_u + A_s H_s)^{(1-\alpha)}$$

so

$$\begin{aligned} H &= A_u H_u + A_s H_s = AH, \\ \text{with } A &= \frac{A_u H_u + A_s H_s}{H} \text{ and } H = H_u + H_s \end{aligned}$$

CC clasify workers into those that didn't finish primary schooling (unskilled) and those that did (skilled). They also explore different classifications.

They assume that workers within each of these groups are perfect substitutes, and aggregate up to total efficiency units of unskilled (H_u) and skilled (H_s) workers using the Mincer approach:

$$
\begin{aligned}
H_u &= \int_{s<6} h(s) dF(s) \\
H_s &= \int_{s\geq 6} h(s) dF(s)
\end{aligned}
$$

with $h(s) = e^{\beta s}$, where β is each country's Mincer coefficient for the relevant range of s from Bils and Klenow (2000).

H_u/H_s differs widely across countries: from 2. 3 in India and 2. 4 in El Salvador to 0.02 in the United States.

In the standard approach, knowing H and K and Y allows one to back-out A from $Y = K^{\alpha}(AH)^{1-\alpha}$.

But now we have two unknowns, A_u and A_s, so we need more information. CC use $w_s/w_u = e^\gamma$ and

$$\frac{w_s}{w_u} = \left(\frac{A_s}{A_u}\right)^\sigma \left(\frac{H_s}{H_u}\right)^{\sigma-1}$$

Using the production function and this equation we get

$$\begin{aligned} A_i &= Y(K/Y)^{-\alpha/(1-\alpha)} m_i^{1/\sigma}/H_i \\ \text{with } m_i &= \frac{w_i H_i}{w_u H_u + w_s H_s} \text{ for } i = s, u \end{aligned}$$

Note that

$$\begin{aligned} A &\equiv \frac{Y}{(K/Y)^{\alpha/(1-\alpha)} H} \\ &= \left(\left(\frac{A_u H_u}{H}\right)^\sigma + \left(\frac{A_s H_s}{H}\right)^\sigma\right)^{1/\sigma} \\ &= A_s m_s^{1-1/\sigma} \frac{H_s}{H} + A_u m_u^{1-1/\sigma} \frac{H_u}{H} \end{aligned}$$

Table 1

Calibration: $\alpha = 1/3$ and $\sigma = 1.4$.

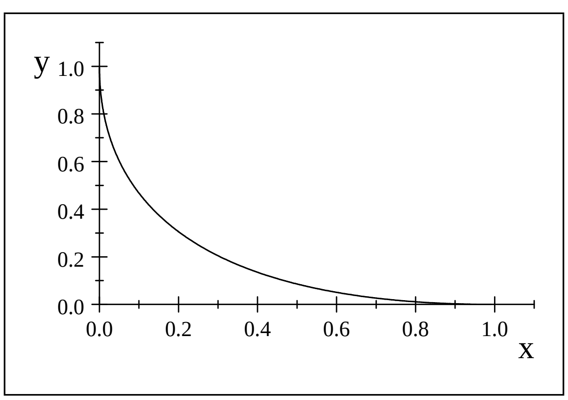
Table 2

Figures 2 - 3

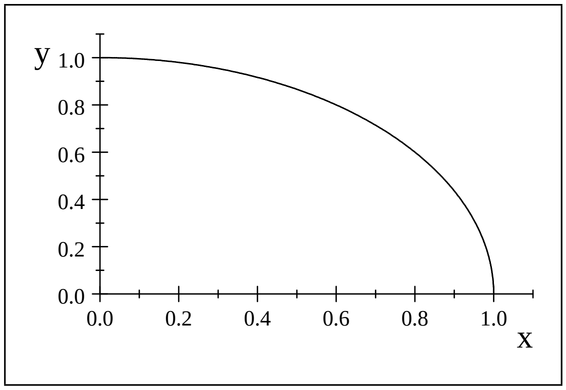
Assume that countries choose H_u, H_s, K and A_u and A_s to maximize profits $(Y - w_u H_u - w_s H_s - rK)$ subject to

$$\begin{aligned} A_s^{\omega} + \gamma A_u^{\omega} &\leq B \\ Y &= K^{\alpha}((A_u H_u)^{\sigma} + (A_s H_s)^{\sigma})^{(1-\alpha)/\sigma} \end{aligned}$$

Two examples: $\gamma = B = 1$ and $\omega = 1/2$:



while $\gamma = B = 1$ and $\omega = 2$:



Note also that if γ is high then the frontier is steep, so that increasing A_u implies a steep decline in A_s.

Easy to see that for interior solution and symmetric equilibrium need $\sigma > \omega$: otherwise, given H_s and H_u, would want to go to corner of $A_s^\omega + \gamma A_u^\omega \leq B$.

But this is not enough, because H_s and H_u are not fixed... need $\omega > \sigma/(1-\sigma)$.

If we use $1/(1-\sigma) = 1.4$ then $\sigma/(1-\sigma) = 0.4$. So need $\omega > 0.4$.

The FOCs are

$$\frac{w_s}{w_u} = \left(\frac{A_s}{A_u}\right)^{\sigma} \left(\frac{H_s}{H_u}\right)^{\sigma-1}$$

and

$$\left(\frac{A_s}{A_u}\right)^{\omega-\sigma} = \gamma \left(\frac{H_s}{H_u}\right)$$

If $\sigma > 0$ (substitutes) and $\omega > \sigma/(1-\sigma)$ then $\omega > \sigma$, hence if H_s/H_u higher then A_s/A_u higher.

CC make the assumption that γ is uncorrelated with endowments (problematic?), take logs, and run a regression

$$\log\left(\frac{A_s^i}{A_u^i}\right) = \frac{\sigma}{\omega-\sigma} \log\left(\frac{H_s^i}{H_u^i}\right) + \frac{1}{\omega-\sigma} \log \gamma^i$$

This yields a regression coefficient of 2.28 with a s.e. of 0.086. If $1/(1-\sigma) = 1.4$ then $\sigma = 0.286$ hence $\omega = 0.41$. Get γ^i from the residual and then back out B^i from $A_s^\omega + \gamma A_u^\omega = B^i$. World technology frontier is the envelope of all these frontiers - World Technology Frontier.

Figured 5 - 7

Calculate A as usual and do decomposition with $y = k^{\alpha}(Ah)^{1-\alpha}$: 40% A and 60% factors.

Correlation between $\log(B)$ and $\log(A)$ is 0.96.

If all countries had access to the World Technology Frontier then the s.d. of $\log y$ would be 0.41 vs. 0.8 in the data - differences in inputs explain about 50% of observed differences in incomes. So 50% B and 50% factors.

We have

$$
\begin{aligned}
A &= \left(\left(\frac{A_u H_u}{H}\right)^{\sigma} + \left(\frac{A_s H_s}{H}\right)^{\sigma}\right)^{1/\sigma} \\
&= \left(\left(\frac{H_u}{H}\right)^{\sigma} + \left(\frac{A_s}{A_u}\frac{H_s}{H}\right)^{\sigma}\right)^{1/\sigma} A_u \\
&= \left(\left(\frac{H_u}{H}\right)^{\sigma} + \left(\gamma^{1/(\omega-\sigma)}\left(\frac{H_s}{H_u}\right)^{1/(\omega-\sigma)}\frac{H_s}{H}\right)^{\sigma}\right)^{1/\sigma} A_u
\end{aligned}
$$

But $A_s^\omega + \gamma A_u^\omega = B$ implies

$$A_u = \left(\frac{B}{(A_s/A_u)^\omega + \gamma}\right)^{1/\omega}$$

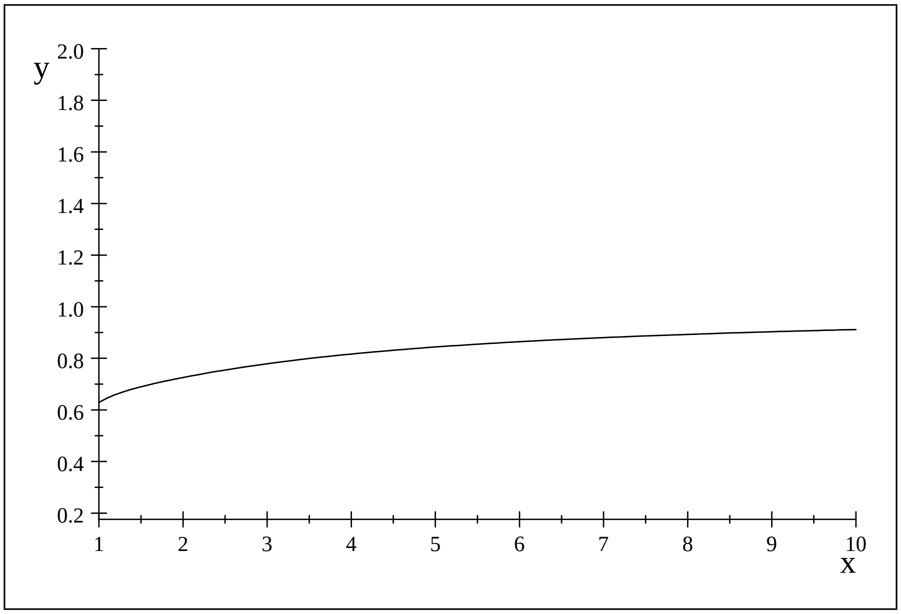
hence

$$\begin{aligned} A &= \left(\left(\frac{H_u}{H}\right)^\sigma + \left(\gamma^{1/(\omega-\sigma)}\left(\frac{H_s}{H_u}\right)^{1/(\omega-\sigma)}\frac{H_s}{H}\right)^\sigma\right)^{1/\sigma} \\ &\quad \cdot \left(\frac{B}{(A_s/A_u)^\omega + \gamma}\right)^{1/\omega} \\ &= (H_u/H)\left(1 + \left(\gamma^{1/(\omega-\sigma)}\left(\frac{H_s}{H_u}\right)^{1/(\omega-\sigma)}\frac{H_s}{H_u}\right)^\sigma\right)^{1/\sigma} \\ &\quad \cdot \left(\frac{B}{\left(\gamma\left(\frac{H_s}{H_u}\right)\right)^{\omega/(\omega-\sigma)} + \gamma}\right)^{1/\omega} \end{aligned}$$

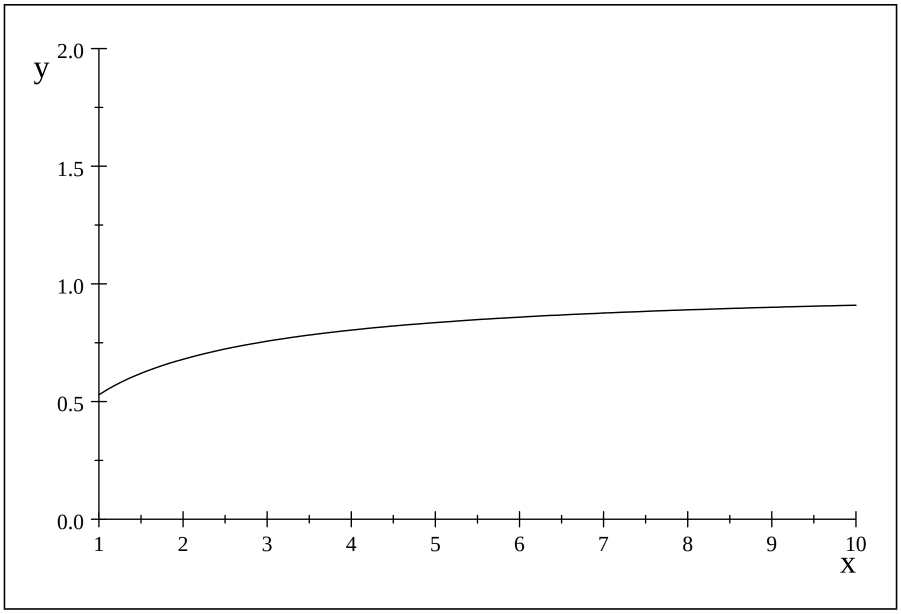
Letting $H_s/H_u = x$ then we have

$$\begin{aligned} A &= \left(\frac{1}{1+x}\right)\left(1 + \left((\gamma x)^{1/(\omega-\sigma)}x\right)^\sigma\right)^{1/\sigma} \\ &\quad \cdot \left(\frac{B}{(\gamma x)^{\omega/(\omega-\sigma)} + \gamma}\right)^{1/\omega} \end{aligned}$$

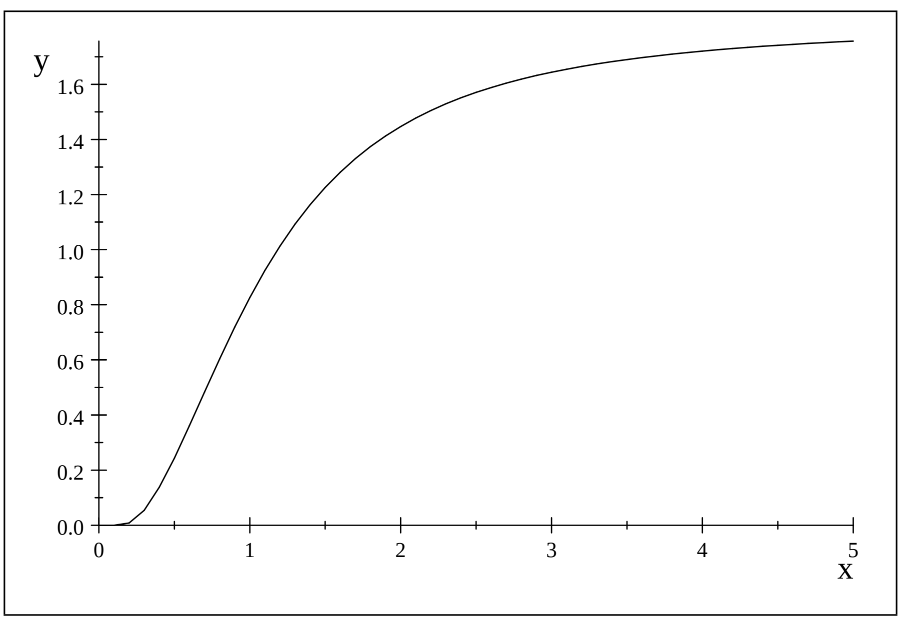
CC estimate $\omega = 0.41$ and use $\sigma = 0.28$. Let's set $B = 1$ and $\gamma = 2$ and plot this curve,



If instead we have $\gamma = 4$ then



Perhaps better to plot $A(x = 10)/A(x = 1)$ against different values of γ,



What is γ for the World Technology Frontier? Have

$$
\begin{aligned}
A_i &= Y(K/Y)^{-\alpha/(1-\alpha)} m_i^{1/\sigma}/H_i \\
\text{with } m_i &= \frac{w_i H_i}{w_u H_u + w_s H_s} \text{ for } i = s, u
\end{aligned}
$$

For Italy have

$$
m_s = \frac{(w_s/w_u)\,H_s}{H_u + (w_s/w_u)H_s} = \frac{1.1 * 66}{43 + 1.1 * 66} 0.63
$$

so

$$
A_s = 29552 * (82318/29552)^{-0.5} 0.63^{1/0.286}/66 = 53.3
$$

$$
A_u = 29552 * (82318/29552)^{-0.5} 0.37^{1/0.286}/43 = 12.73
$$

Hence

$$
\begin{aligned}
\log \gamma &= (\omega - \sigma) * \log\left(\frac{A_s}{A_u}\right) - \sigma \log\left(\frac{H_s}{H_u}\right) \\
&= (0.41 - 0.286) * \log\left(\frac{53.3}{12.73}\right) - 0.286 * \log\left(\frac{66}{43}\right)
\end{aligned}
$$

$$
\begin{aligned}
\gamma &= \exp\left((0.41 - 0.286) * \log\left(\frac{53.3}{12.73}\right) - 0.286 * \log\left(\frac{66}{43}\right)\right) \\
&= 1.057
\end{aligned}
$$

Thus, if anything, there is even larger TFP differences than we estimate from the standard method.