Econ 580, Lecture Notes 8

Andres Rodriguez-Clare

Penn State University

February 26, 2009

Endogenous Growth a la Romer (1990) and quasi-endogenous growth a la Jones (1995)

R&D = coming up with new ideas to produce goods - higher quality, or new varieties.

Romer (1990): new varieties.

Aghion and Howitt, or Grossman and Helpman: quality ladder models.

An idea is not an object: it is not affected by scarcity, it can be used to produce 1 units or 1,000 without any decreasing returns setting in.

This is called non-rivalry.

Note that under CRS (replicability) and perfect competition, there is nothing left to pay inventors... imperfect competition is inevitable.

Alternative views: rival "knowledge" capital.

Endogenous growth with perfect competition and CRS - the "AK" model.

We think of innovation as investing in R&D to come up with new varieties of an intermediate good.

Given love of variety then this increases utility or productivity.

CES production function with elasticity $\sigma = 1/(1-\alpha)$,

$$X = \left(\int_0^\infty z(j)^\alpha dj \right)^{1/\alpha}$$

At any point in time there will be some intermediate goods that have been invented. Let's order goods so that $j \in [0, n(t)]$ are available at time t.

We will drop the time index from now on unless necessary to avoid confusion.

The only factor of production is labor, which is used to produce z with a CRS one-to-one technology.

L is supplied inelastically and is constant through time. We use labor as the numeraire so $w = 1$.

Note that in static models of monopolistic competition $z(j)$ is produced with a fixed cost and a constant marginal cost, and n is determined by the ZPC.

Now there is no fixed cost, n is a state variable, and the ZPC applies to R&D to determine $\dot{n}$ in equilibrium (below).

Labor can be allocated to produce intermediate goods already invented or to R&D.

We look for a steady state where the share of labor devoted to producing intermediate goods, β, is fixed. The fraction $1 - \beta$ of L is devoted to R&D. Then the total units sold by each monopolist of intermediate goods is $z = \beta L/n$.

The price at which these units will be sold is $p_z = w/\alpha = 1/\alpha$. Hence, the profit flow is ($\theta \equiv (1-\alpha)/\alpha$)

$$\pi(n) = p_z z - wz = \theta \beta L/n$$

Total expenditure on intermediate goods will be constant and equal to $p_z z n = \beta L/\alpha$.

In equilibrium, the value of a new patent must equal the cost of generating that patent. We look for a steady state with constant growth rate of n, $g = \dot{n}/n$.

The value of a new patent is the present discounted sum of profits, taking into account the interest rate r and the fact that n is growing at g. This implies that

$$\begin{aligned} V(n(t)) &= \int_t^\infty \pi(n(s)) e^{-rs} ds \\ &= \frac{\theta \beta L}{n(t)} \int_t^\infty e^{-(r+g)s} ds \\ &= \frac{\pi(n(t))}{r+g} \end{aligned}$$

Note that a high g reduces V because as n increases profits decline.

But what is r? We need to specify intertemporal preferences. Assume that

$$U(t) = \int_t^\infty \mathrm{ln}(c(s))e^{-\rho(s-t)}ds$$

where c is the consumption of good X above. The price of X is simply

$$p(n) = \left(\int_0^n p_z(j)^{1-\sigma}dj\right)^{1/(1-\sigma)} = n^{1/(1-\sigma)}/\alpha$$

Consumers maximize utility subject to their intertemporal budget constraint,

$$\int_t^\infty c(s)p(n(s))e^{-r(s-t)}ds = \int_t^\infty e^{-r(s-t)}ds$$

Let $E(t)$ be the expenditure by consumers at time t. We have $E(t) = c(t)p(t)$. Then the optimization problem is simply to maximize

$$\int_t^\infty \left[\mathrm{ln}(E(s)) - \mathrm{ln}(p(s))\right] e^{-\rho(s-t)}ds + \int_t^\infty \lambda[1 - E(s)]e^{-r(s-t)}ds$$

where λ is a scalar Lagrange multiplier.

The F.O.C. is

$$\frac{e^{-\rho(s-t)}}{E(s)} = \lambda e^{-r(s-t)}$$

which implies that

$$\dot{E}/E = r - \rho$$

But since the wage is fixed, E must be constant, and hence this implies that $r = \rho$.

We now have

$$V(n) = \frac{\pi(n)}{\rho + g}$$

What is the cost of coming up with a patent? Assume that

$$\dot{n} = K(n)L_I/a_I$$

where $K(n)$ is "knowledge capital."

The cost of a patent is simply $a_I/K(n)$, and the equilibrium condition for R&D is

$$\left(\frac{1}{\rho + g}\right)\left(\frac{\theta\beta L}{n}\right) = \frac{a_I}{K(n)}$$

If $K(n)$ is constant then there is no steady state with positive R&D.

Assume instead that $K(n) = n$ (knowledge spillovers in R&D).

Then

$$\left(\frac{1}{\rho + g}\right)\left(\frac{\theta\beta L}{n}\right) = \frac{a_I}{n}$$

which implies

$$\theta\beta L = a_I(\rho + g)$$

The only thing left is to determine β. This is simply done by noting that $g = \dot{n}/n$, $\dot{n} = nL_I/a_I$, and $L_I = (1 - \beta)L$, so

$$g = (1 - \beta)L/a_I$$

Together these last two equations imply

$$g(L) = \frac{(1 - \alpha)L}{a_I} - \alpha\rho$$

The formula for the growth rate,

$$g(L) = \frac{(1-\alpha)L}{a_I} - \alpha\rho$$

implies that there is a "strong scale effect" – higher L leads to higher g.

This is not consistent with the data (Jones, 1995). One solution: quasi-endogenous growth

Assume that $K(n) = n^{\gamma}$, with $\gamma < 1$. Then for a steady state we need

$$\left(\frac{\theta\beta/a_I}{\rho+g}\right) L = n^{1-\gamma}$$

Taking logs and differentiating implies that

$$\begin{aligned} g_L &= (1-\gamma)\, g \\ &\Longrightarrow \; g = \frac{g_L}{1-\gamma} \end{aligned}$$