Econ 580, Lecture Notes 9

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In the standard development accounting exercise we think of A as exogenous at the country level.

We now explore how results change when A is endogenous to "research" that varies across countries

Paper: Cordoba and Ripoll, 2008

A final good is produced via a CES as in Romer (1990), with elasticity $\varepsilon \equiv 1/(1-\gamma)$,

$$Y = \left(\int_0^\infty x(j)^\gamma dj \right)^{1/\gamma}$$

(notation is different than in CR)

At any point in time there will be some intermediate goods that have been invented in the world, N_t^*

Some but not all of these goods will be available in our country of interest, $N_t < N_t^*$

N_t^* grows exogenously at rate g

In steady state, N_t will be some constant (endogenous) fraction of N_t^*

Available intermediate goods are produced from physical and human capital according to

$$x(j) = k(j)^{\alpha} h(j)^{1-\alpha}$$

where

$$\int_0^{N_t} k(j)dj = K_t \text{ and } \int_0^{N_t} h(j)dj = H_t = h_t L_t$$

Output can be used for consumption, investment or "research,"

$$Y_t = C_t + I_t + R_t$$

Research entails adoption of foreign varieties,

$$\dot{N}_t = (R_t/L_t) \cdot (N_t^*/N_t)^{\eta} \cdot N_t^{\upsilon}$$

Three things to note:

1. Benefits of backwardness: lower N_t/N_t^* implies higher $\dot{N}_t$ (positive global externalities)

2. Positive country-level externalities if $\upsilon > 0$... but will need $\upsilon < 0$ for a steady state

3. Economies of scale are killed by having R_t divided by L_t

Think of cost (in units of final good) of adopting a new variety as

$$\lambda_t = N_t^{-\phi} (N_t^*)^{-\eta} L_t$$

where $\phi = \upsilon - \eta$. To determine $s_R \equiv R/Y$ in the steady state equilibrium we need $\lambda_t = V_t$

When a variety is adopted, researcher gets a patent which may expire at any point at rate p so $\dot{N}_{mt} = \dot{N}_t - pN_{mt}$ and $\dot{N}_{ct} = pN_{mt}$ and in steady state we have

$$N_{mt}/N_{ct} = g/p$$

If π_t are the profits realized by monopolists at time t then

$$V_t = \int_t^{\infty} \pi_v e^{-(r+p)(v-t)} dv$$

We need to derive π_v and r

Finally, authors introduce a confiscation probability q (distributed back to consumers lump-sum) so that the no-arbitrage condition is $1 + r_t = (1 + r_{kt} - \delta)(1 - q)$, and hence

$$r_{kt} = \frac{r_t + q + \delta(1 - q)}{1 - q} \equiv f(r, q)$$

The policy variables are then going to be p (which affects incentives to innovate) and q (which affects incentives to accumulate physical capital)

A preview with simpler model

Imagine that $p = q = 0$. Then it is easy to show that

$$Y = N^{1/(\varepsilon-1)} K^{\alpha} H^{1-\alpha}$$

This is the standard love of variety.

A fraction $1-\gamma$ of output would go to holders of patents, so

$$\pi = \frac{(1-\gamma)Y}{N}$$

while a fraction γY goes to pay capital and labor, with $r_k = r + \delta = \gamma\alpha Y/K$ and $wH = \gamma(1-\alpha)Y$.

We now have

$$Y = N^{\beta} X L$$

where $X = (K/Y)^{\alpha/(1-\alpha)} h$ and

$$\beta = \frac{1}{\varepsilon - 1} \cdot \frac{1}{1-\alpha}$$

What is r? The intertemporal elasticity of consumption of σ and discount rate ρ imply that

$$g_c = \frac{\dot{c}_t}{c_t} = \sigma(r_t - \rho)$$

In steady state $g_c = g_y = g_k = \beta g$, so now we have pinned down r in terms of parameters,

$$r = \beta g/\sigma + \rho$$

Is there a steady state? In steady state with a positive $\dot{N}/N$ we need

$$\frac{\dot{N}}{N} = \frac{(R/L)\,(N^*/N)^\eta\,N^\upsilon}{N} = s_R y N_t^{\phi-1}\,(N_t^*)^\eta = g$$

Log differentiation implies that

$$\beta g + (\phi - 1)g + \eta g = 0$$

For $g > 0$ this entails $\beta + \upsilon = 1$, so we need **Assumption 1**:

$$\upsilon = 1 - \beta$$

(Note that $\upsilon < 0$). This may seem very restrictive. In Howitt (2000) and KRC (2005) this kind of knife edge condition is not needed because they have the equivalent to

$$\dot{N}_t = (R_t/L_t) \cdot (1 - N_t/N_t^*)$$

Finally, we need to solve for s_R and N. Differentiation of $\lambda_t = V_t = \int_t^\infty \pi_v e^{-r(v-t)} dv$ and rearranging yields the no-arbitrage condition,

$$\pi_t/\lambda_t + \dot{\lambda}_t/\lambda_t = r$$

But $\pi = (1-\gamma)Y/N$, $R_t = \lambda_t \dot{N}_t$ and $g_N = g$ imply that

$$\frac{\pi}{\lambda} = \frac{(1-\gamma)YR\dot{N}}{\lambda NR\dot{N}} = \frac{(1-\gamma)g}{s_R}$$

whereas $\lambda_t = N_t^{-\phi}\left(N_t^*\right)^{-\eta} L_t$ implies that $g_\lambda = -(\phi + \eta)g + g_L$, hence

$$s_R = \frac{(1-\gamma)g}{r + (\phi+\eta)g - g_L}$$

We cannot use $y = N^\beta X$ as a decomposition because given s_R then X affects N. In particular, using assumption 1 then from $\dot{N} = (R/L)\left(N^*/N_t\right)^\eta N_t^\upsilon$ and $g_N = g$ we find that $(N_t/N_t^*)^\eta = s_R X/g$.

But we can use $(N_t/N_t^*)^\eta = s_R X/g$ to write

$$y = \left(\frac{s_R X}{g}\right)^{\beta/\eta} (N_t^*)^\beta X$$

which can be rewritten as $y_t = \widehat{A}_t \widehat{X}$ where

$$\widehat{A}_t \equiv (s_R/g)^{\beta/\eta} (N_t^*)^\beta \text{ and } \widehat{X} \equiv X^{1+\beta/\eta}$$

Comments:

1. Once TFP is endogenous, then X affects TFP... standard decomposition is not valid

2. The term $\widehat{X}$ in the new decomposition has the added elasticity β/η that captures the effect of X on TFP

3. The term β in β/η is $\frac{1}{\varepsilon-1}\cdot\frac{1}{1-\alpha}$, L.O.V. and standard capital amplification of TFP differences

4. The division by η in β/η comes from the negative effect of an increase in N/N^* on $\dot{N}$

Calibration and Results

Use rate of conditional convergence to calibrate β/η.

They actually use a Solow model with exogenous (and constant) s and s_R and

$$Y = K^{\alpha}(A^{\beta}H)^{1-\alpha}$$

and $\dot{H} = g_L$ and

$$\dot{A} = \left(\frac{A^*}{A}\right)^{\eta} A^{\upsilon} R$$

but set $\beta = 1$ and $\upsilon = 0$.

The speed of convergence is ζ where

$$\log\left[\widehat{y}(t)\right] = e^{-\zeta t} \cdot \log\left[\widehat{y}(0)\right] + (1 - e^{-\zeta t}) \log(\widehat{y}^*)$$

where $\widehat{y} = y/H$.

In the Solow model the speed of convergence is

$$\zeta = \psi \equiv (g_L + g_y + \delta)(1 - \alpha)$$

With $g_y = 0.015$ and $g_L = 0.011$ and $\delta = 0.08$ and $\alpha = 1/3$ this yields $(0.08 + 0.011 + 0.015) * (2/3) = 0.07$ or 7%, which is significantly higher than the range of $2 - 4\%$ in most estimates of β.

In the CR model... log linearization of the model around the steady state yields the following characteristic equation:

$$x^2 + (\psi + \eta g_y + \alpha g_y)\, x + \psi \eta g_y = 0$$

We use $g_y = 0.015$ and $\alpha = 1/3$. Hence $\alpha g_y \approx 0$ and we can approximate the equation by

$$(x + \psi)\,(x + \eta g_y) = 0$$

We know that ζ is determined by the smaller root (in absolute value) of this equation (eigenvalue). To get a ζ close to 4 we need $\eta g_y = 0.04$, hence $\eta = 0.04/0.015 = 2.7$, hence $\beta/\eta = 1/2.7 = 0.38$. Alternatively for a $\zeta = 0.03$ we need $\beta/\eta = 0.015/0.03 = 0.5$.

Table 2. Variance Decomposition - Extended Solow Model

	% contribution to variance of $\log(Y/L)$				
	Factors of production			TFP	Covariance
Model	$\kappa^{\frac{\alpha}{1-\alpha}(1+\beta/\eta)}$	$h^{(1+\beta/\eta)}$	$\widehat{X}$	$\widehat{A}$	$cov(\widehat{X},\widehat{A})$
$\beta/\eta = 0.00$	17	23	40	60	35
$\beta/\eta = 0.38$	23	32	55	45	25
$\beta/\eta = 0.50$	25	35	60	40	19
$\beta/\eta = 0.60$	27	37	64	36	13

Complete Model

The authors now want to push this back so that instead of talking about endogenous (although economically independent) variables $\widehat{A}_t$ and $\widehat{X}$, we talk about exogenous policy variables p and q

Intermediate goods are produced with physical and human capital at unit cost

$$e_t = r_{kt}^{\alpha} w_t^{1-\alpha}$$

and $P_{mt} = e_t/\gamma$ while $P_{ct} = e_t$. We have to deal with the fact that intermediate goods vary in price, so that

$$x_m = x_c/a_1, \quad a_1 = \gamma^{-1/(1-\gamma)}$$

Using $x_m N_m + x_c N_c = K^{\alpha} H^{1-\alpha}$ then we get

$$x_m = \frac{K^{\alpha} H^{1-\alpha}}{N_m} \cdot \frac{N_m}{N_m + a_1 N_c}$$

The term

$$\frac{N_m}{N_m + a_1 N_c}$$

is the fraction of total costs of intermediate goods devoted to producing goods with patents

Some further manipulation yields

$$\pi_t = \frac{(1-\gamma)Y_t}{N_m + a_1^{\gamma} N_c} = \frac{(1-\gamma)Y_t}{N_m} \cdot \frac{N_m}{N_m + a_1^{\gamma} N_c}$$

This is the part of output that goes to patent holders. The second term can be shown to be the share of total revenues by producers of intermediate goods that goes to patent holders:

$$\frac{P_m x_m N_m}{P_m x_m N_m + P_c x_c N_c} = \frac{N_m}{N_m + a_1^{\gamma} N_c}$$

Another way to write this is recall that $\pi_t N_{mt} = (1 - 1/m)Y_t$, where m is the average mark-up, and is now given by

$$\begin{aligned} m &= \frac{P_m x_m N_m + P_c x_c N_c}{e x_m N_m + e x_c N_c} \\ &= \frac{1}{\gamma}\left(\frac{N_{mt} + a_1^{\gamma} N_{ct}}{N_{mt} + a_1 N_{ct}}\right) \end{aligned}$$

Note that if $p \to \infty$ (no patents) then $m \to 1$ while if $p \to 0$ then $m \to 1/\gamma$. Plugging this into $\pi_t N_{mt} = (1 - 1/m)Y_t$ yields the result above, namely

$$\pi_t = \frac{(1-\gamma)Y_t}{N_m + a_1^{\gamma} N_c}$$

Note that whereas profits when $p = 0$ were $(1-\gamma)Y/N$, now they are $(1-\gamma)Y/\left(N + (a_1^{\gamma} - 1)N_c\right)$. Since $a_1^{\gamma} > 1$ then, given Y, profits per monopolist decrease as p increases. The intuition is that competition leads to lower prices for N_c varieties, and this reduces the quantity sold by monopolists. In other words, an increase in N_c makes competition tougher also for patent holders, so their profits decline. For future reference, note that profits decline relative to $N_c = 0$ by a factor

$$\frac{N_m + N_c}{N_m + a_1^\gamma N_c} = \frac{g + p}{g + a_1^\gamma p} < 1$$

From $Y = (\int_0^\infty x(j)^\gamma dj)^{1/\gamma}$ we get (note: $1/(\varepsilon - 1) = (1-\gamma)/\gamma$)

$$Y = N^{(1-\gamma)/\gamma} \cdot \frac{\left(n_m + a_1^\gamma (1 - n_m)\right)^{1/\gamma}}{n_m + a_1 (1 - n_m)} \cdot K^\alpha H^{1-\alpha}$$

where $n_j = N_j/N$. The second term is lower than one and captures the loss associated with the mark-up distortion

Also,

$$r_{kt}K_t = \gamma \frac{N_{mt} + a_1 N_{ct}}{N_{mt} + a_1^{\gamma} N_{ct}} \alpha Y_t$$

To understand this, note that $(1 - 1/m)$ is the part of output that goes to patent holders while $1/m$ is the part that goes to physical and human capital, so

$$r_{kt}K_t = \frac{\alpha Y_t}{m}$$

Note that if $p \to \infty$ (no patents) then $r_{kt}K_t = \alpha Y_t$.

In general, using $r_{kt} = f(r_t, q)$, and

$$m = \frac{1}{\gamma}\left(\frac{N_{mt} + a_1^{\gamma} N_{ct}}{N_{mt} + a_1 N_{ct}}\right) = \frac{1}{\gamma}\left(\frac{g + a_1^{\gamma} p}{g + a_1 p}\right) \equiv m(p)$$

we get

$$K/Y = \kappa(p, q) = \frac{\alpha}{m(p)} \cdot \frac{1}{f(r, q)}$$

with

$$r = \beta g/\sigma + \rho$$

Note that both policies, p and q, affect K/Y.

- The effect of q is obvious.
- As to p, note that $a_1 > 1$ and $0 < \gamma < 1$, hence $a_1^\gamma < a_1$, so an increase in p lowers m and hence increases K/Y.

In the simple model we had

$$s_R = \frac{(1-\gamma)g}{r + (\phi+\eta)g - g_L}$$

Now the discount rate is increased by p, but profits per monopolist are lower than if $N_c = 0$ (see above), so

$$s_R(p) = \left(\frac{(1-\gamma)g}{r + p + (\phi+\eta)g - g_L}\right)\left(\frac{g+p}{g + a_1^\gamma p}\right)$$

The result

$$Y = \frac{\left[N_m + a_1^\gamma N_c\right]^{1/\gamma}}{N_m + a_1 N_c} K^\alpha H^{1-\alpha}$$

can be simplified to

$$Y_t = K_t^\alpha (A_t^\beta H_t)^{1-\alpha}$$

where

$$A_t \equiv \chi(p) N_t,\ \beta \equiv \frac{1}{\varepsilon - 1} \cdot \frac{1}{1-\alpha}, \text{ and}$$

$$\chi(p) \equiv \left[\left(\frac{g + a_1^\gamma p}{g+p}\right)^{1/\gamma} \frac{g+p}{g+a_1 p}\right]^{\frac{\gamma}{1-\gamma}}$$

Finally, we have

$$
\begin{aligned}
\widehat{X} &= \widehat{X}(q,p,h) = \left(\kappa(p,q)^{\alpha/(1-\alpha)} h\right)^{1+\beta/\eta} \\
\widehat{A} &= \widehat{A}(p) = \left(s_R(p)\chi(p)^{1-\phi}\right)^{\beta/\eta} (N^*)^{\beta}
\end{aligned}
$$

and so

$$
y = \widetilde{A}(p) \cdot \widetilde{X}(q,h)
$$

where

$$
\widetilde{A}(p) = \Omega(p)\widehat{A}(p) \quad \text{and} \quad \widetilde{X}(q,h) = \Omega(p)^{-1}\widehat{X}
$$

and

$$
\Omega(p) = \left(\frac{g + a_1 p}{g + a_1^{\gamma\gamma} p}\right)^{\frac{\alpha}{1-\alpha}(1+\beta/\eta)}
$$

Table 3. Variance Decomposition - Varieties Microfounded Model

	% contribution to variance of $\log(Y/L)$		
Model	$\widehat{X}$	$\widehat{A}$	$cov(\widehat{X}, \widehat{A})$
$\beta/\eta = 0.00$	41	59	35
$\beta/\eta = 0.38$	56	44	26
$\beta/\eta = 0.50$	61	39	20
$\beta/\eta = 0.60$	65	35	15