

Econ 580, Homework on PPP exchange rates

To understand the importance of using "international dollars" (Penn World Table) rather than "U.S. dollars" (regular GDP data, transformed into a common currency at regular exchange rates), think about the following scenario.

There is only labor.

There are two goods: a non-tradable good (e.g., haircuts) denoted by x and a tradable good (e.g., apples) y .

Let A_i^x and A_i^y be the labor productivity in country i in x and y .

Imagine that $A_i^x = 1$ in all countries, but A_i^y differs across countries (i.e., it is high in rich countries relative to poor countries).

Assume that the domestic currency or unit of account in each country is such that the wage is equal to 1.

Questions:

1. What is the exchange rate across any two countries?
2. Think of country $i = 1$ as the U.S., and let's call the U.S. unit of account the "US dollar." What is the US dollar cost of y in country i ?
3. What is the US dollar cost of x in country i ? How does it differ across rich and poor countries?
4. Imagine that preferences are Cobb-Douglas, with ϕ equal to the share of income spent on x . (NOTE: perhaps a better interpretation is that x and y are intermediate goods, which as used to produce a non-tradable consumption good with a CD production function with share ϕ for x .) What is the correct price index in units of the domestic unit of account for country i in terms of A_i^y ? Denote this price index in country i by p_i . Note that since the wage in country i in terms of its domestic unit of account is 1, then the utility level in country i is simply $1/p_i$.
5. Define an "international dollar" as a unit of "currency" that has the same purchasing power everywhere, meaning that spending a unit of each currency in any country delivers the same level of utility. Furthermore, assume that let's normalize things so that exchange rate between the US dollar and the international dollar is one. Let E_i^{PPP} be the price of the domestic currency of country i in terms of international dollars. These are called PPP (for purchasing power parity) exchange rates. Calculate an expression for E_i^{PPP} .
6. What is country i 's GDP valued at domestic prices?
7. Derive an expression for the ratio of GDP in international dollars to GDP in US dollars. How does this relate to A_i ? What does this say about the measurement of GDP in US dollars (i.e., using market exchange rates) and international dollars (i.e., using PPP exchange rates)?

8. A useful approximation to E_i^{PPP} is

$$\tilde{E}_i^{PPP} = \phi + (1 - \phi)A_i/A_1$$

(Why do we say that this is an approximation to E_i^{PPP} ?) Define "international prices" as the prices in international dollars for goods purchased in the US. Show that for any country i , GDP valued at international prices is the same as GDP in international dollars.

NOTE: this PPP exchange rate in point 5 and not the one in point 1 is the one used in the Penn World Table. The terminology is that the GDP levels calculated in this way are expressed at "purchasing power parity" - the idea is that if you have some amount of US dollars (or any other currency), and exchange them for the currency of country i at the PPP exchange rate, then you can buy the same number of utils that you would have bought in the U.S.