The Simple Analytics of the Melitz Model in a Small Economy*

Svetlana Demidova†
McMaster University

Andrés Rodríguez-Clare‡
University of California at Berkeley and NBER

August 10, 2011

Abstract

In this paper we present a version of the Melitz (2003) model for the case of a small economy and summarize the key relationships in the model in a simple picture. We then show how this approach helps to provide a simple and intuitive analysis of the implications of asymmetric changes in trade barriers. In particular, we show that a decline in the costs of importing and/or exporting increases welfare of the liberalizing country.

JEL classification: F12, F13

Key words: firm heterogeneity, small economy, trade liberalization

1 Introduction

In this paper we present a version of the Melitz (2003) model for the case of a small economy. We show that unlike the case of the Melitz (2003) setup with large economies, the equilibrium analysis can be carried out with the help of a simple figure that summarizes the key relationships in the model. In particular, we show that the equilibrium can be fully characterized by two conditions that relate the wage with the productivity cut-off for exporters in the small country. First, there is a "competitiveness" condition, according to which a higher wage reduces the country's competitiveness, and this leads to an increase in the productivity cut-off for exporting. Second, there is a "trade balance" condition, according to which an increase in the productivity cut-off for exporting leads to a decline in exports and, hence, a trade deficit. The deficit must be counteracted by a decline in the wage, which increases exports and decreases imports. These two conditions give us two curves, the *competitiveness curve* and the *trade balance curve*, one sloping upwards and one downwards as shown in Figure 1, and their intersection gives the equilibrium.

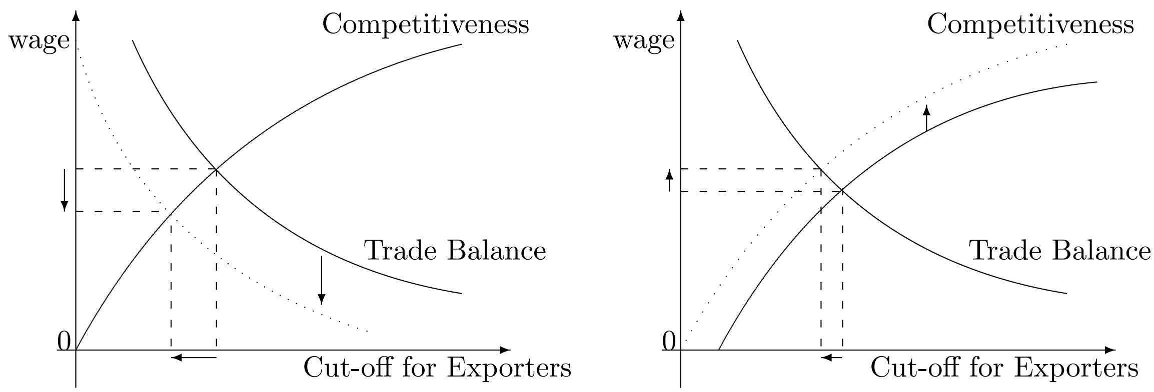
We illustrate the usefulness of this approach by exploring the implications of asymmetric changes in trade barriers. With the aid of our simple figure, we show that unilateral trade liberalization

*We thank Arnaud Costinot and Kala Krishna for helpful comments. All remaining errors are our own.

†Department of Economics, McMaster University, Hamilton, ON, Canada L8S 4M4. E-mail: demidov@mcmaster.ca. Phone: (905) 525 9140, ext. 26095. Fax: (905) 521 8232.

‡Department of Economics, University of California at Berkeley. E-mail: andres@econ.berkeley.edu.

Figure 1: The Equilibrium Conditions



(a) Decline in Costs of Importing (b) Decline in Costs of Exporting

(i.e., a decline in the variable or fixed costs of importing) by the small economy does not affect the competitiveness curve but it shifts the trade balance curve downwards, since a lower wage is needed to restore trade balance after imports become cheaper. As we see in Figure 1(a), this leads to a decline in the wage and a decline in the productivity cut-off for exporters. The effect on the real wage is unambiguous: we show that welfare always moves in the opposite direction as the productivity cut-off for exporting, thus, implying that unilateral trade liberalization increases welfare (i.e., the price index falls by more than the wage).[1] Similarly, a decline in the variable cost of exporting leads to a shift up in the competitiveness curve with no movement in the trade balance curve, implying from Figure 1(b) an increase in the wage and also a decline in the productivity cut-off for exporting. Hence, welfare also increases.

In contrast to several recent contributions (e.g., Grossman, Helpman and Szeidl (2006), Demidova (2008), Chor (2009), Baldwin and Okubo (2009), and Baldwin and Forslid (2010) among the others), we do not assume the existence of an "outside" sector that pins down the wage, so our analysis takes into account the effect of trade liberalization on the equilibrium wage. Our model is similar to Demidova and Rodríguez-Clare (2009), but here our focus is different: instead of characterizing the optimal policies to deal with the various distortions in the model, here we show that the model admits a simple and intuitive analysis of the equilibrium determination and comparative statics.

We start in Section 2 by considering the standard case of two large economies. There we show that unilateral trade liberalization by one of these economies moves both the competitiveness and the trade balance curves and so the graphical analysis is insufficient. Changes in the wage and the productivity cut-offs of the liberalizing economy affect the intensity of competition in the other

[1] In the text below we show that the free entry condition implies that the productivity cut-offs for domestic production and for exporting move in opposite directions, and also that the productivity cut-off for domestic production is a sufficient statistic for welfare. A direct implication is that a decline in the productivity cut-off for exporting leads to an increase in welfare.

economy, and this is what leads to the shift in the competitiveness curve. In Section 3 we show that this is no longer true in the case of a small economy, which we show to be the limit of the regular model as one of the countries becomes small.

2 Case of a Large Economy

To demonstrate the advantage of our approach, we will first look at the general Melitz (2003) model of two large but possibly asymmetric economies. We will show how complicated the analysis of comparative statics in this setting can be by looking at the case of unilateral trade liberalization.

2.1 Model

Consider two countries indexed by $i = 1, 2$ and populated by L_i identical households, each of which has a unit of labor supplied inelastically. There is a continuum of goods indexed by $\omega \in \Omega$. The representative consumer has Dixit Stiglitz preferences in each country with elasticity of substitution $\sigma > 1$.

Each country has an (endogenous) measure M_i^e of monopolistically competitive firms that pay a fixed cost $w_i F_i$ to enter the market and draw their random productivity z from the cumulative distribution function $G_i(z)$. Given z, a firm from country i faces a cost w_i/z of producing one unit and decides whether to sell in the domestic market and/or export abroad. Firms from i have to pay a fixed "marketing" cost $w_i f_{ij}$ to sell in market j. Iceberg trade costs are $\tau_{ij} > 1$ so that for a firm in i with productivity z the cost of producing and selling one unit in j is $w_i \tau_{ij}/z$. We assume that $\tau_{ii} = 1$ for $i = 1, 2$.

2.2 Characterization of the Equilibrium

Since profits are monotonically increasing in productivity, z, there is a productivity cut-off z_{ij}^* such that, among country i' firms, only those with a productivity of at least z_{ij}^* decide to sell in market j. Letting $\rho \equiv 1 - 1/\sigma$, these cut-offs are defined implicitly by[2],

$$w_j L_j P_j^{\sigma-1} \left(w_i \tau_{ij} / \rho z_{ij}^* \right)^{1-\sigma} = \sigma w_i f_{ij}, \tag{1}$$

where P_j is the price index in country j given by

$$P_j^{1-\sigma} = \sum_{i=1}^{2} M_i^e \int_{z_{ij}^*}^{\infty} \left(\frac{w_i \tau_{ij}}{\rho z} \right)^{1-\sigma} dG_i(z). \tag{2}$$

The free entry condition for firms in country i equalizes the expected profits of entering the market to the entry costs. Following Melitz (2003), we let $J_i(a) \equiv \int_a^\infty \left[\left(\frac{z}{a} \right)^{\sigma-1} - 1 \right] dG_i(z)$ and

[2] In establishing these conditions for the cut-offs, we have used four standard results. First, firms set prices equal to unit cost multiplied by the mark-up $1/\rho$. Second, firms' variable profits are revenues divided by σ. Third, revenues in market j given a price p are $R_j P_j^{\sigma-1} p^{1-\sigma}$, where R_j are total expenditures in j. And fourth, $R_j = w_j L_j$ since due to free entry the only source of national income is labor payments.

note that (from the definition of the cut-offs z_{ij}^*) the expected profits for country i' firms in country j are $w_i f_{ij} J_i(z_{ij}^*)$. Then the free entry condition in country i is

$$\sum_{j=1}^{2} f_{ij} J_i\left(z_{ij}^*\right) = F_i. \tag{3}$$

Next, let us look at the labor market clearing condition that equalizes total labor demand given by $M_i^e F_i + \sum_{j=1}^{2} L_{ij}$ to labor supply in country i. Using (3), it can be written as

$$M_i^e \sigma \sum_{j=1}^{2} f_{ij}\left[J_i\left(z_{ij}^*\right) + 1 - G_i\left(z_{ij}^*\right)\right] = L_i. \tag{4}$$

Total sales by firms from i in j are

$$X_{ij} = M_i^e \sigma w_i f_{ij} \int_{z_{ij}^*}^{\infty} \left(z/z_{ij}^*\right)^{\sigma-1} dG_i(z) = M_i^e \sigma w_i f_{ij}\left[J_i(z_{ij}^*) + 1 - G_i(z_{ij}^*)\right].$$

Trade balance implies that for $i \neq j$ we have $X_{ij} = X_{ji}$. Hence,

$$M_i^e w_i f_{ij}\left[J_i(z_{ij}^*) + 1 - G_i(z_{ij}^*)\right] = M_j^e w_j f_{ji}\left[J_j(z_{ji}^*) + 1 - G_j(z_{ji}^*)\right]. \tag{5}$$

To summarize, there are 10 unknown equilibrium variables: M_i^e, z_{ii}^*, z_{ij}^*, P_i, and w_i for $i, j = 1, 2$. We have 9 equilibrium conditions: two free entry conditions, four cut-off conditions, two price index equations, and trade balance. Setting labor in one of the countries as numeraire, we can then use the equilibrium conditions to solve for all the unknown variables.[3]

For future reference, we note here that, as in Melitz (2003), welfare in country i rises with the productivity cut-off for domestic sellers, z_{ii}^*. Free entry implies that there are no profits, so the real wage, w_i/P_i, measures welfare in our simple economy. Note that (1) directly implies that

$$\frac{w_i}{P_i} = \left(\frac{L_i}{\sigma f_{ii}}\right)^{\frac{1}{\sigma-1}} \rho z_{ii}^*.$$

As a result, to know what happens to welfare as a result of trade liberalization, we just need to see what happens to the domestic productivity cut-off, z_{ii}^*.

2.3 Graphical Analysis

First, let us normalize wage in country 2 to unity, $w_2 \equiv 1$. Then we can reduce the system of 9 equilibrium conditions with 9 unknowns to 2 equations with 2 unknowns, namely, w_1 and z_{12}^*. To see this, note that from (1) we get

$$z_{12}^* = h_{12}(w_1, z_{22}^*) \equiv \tau_{12}\left(\frac{f_{12}}{f_{22}}\right)^{\frac{1}{\sigma-1}} (w_1)^{\frac{1}{\rho}} z_{22}^*, \tag{6}$$

[3] As is standard in the literature, we assume that iceberg trade and fixed marketing costs are such that $z_{ii}^* < z_{ij}^*$ for all $i, j = 1, 2$.

$$z_{21}^{*}=h_{21}(w_1,z_{11}^{*})\equiv\tau_{21}\left(\frac{f_{21}}{f_{11}}\right)^{\frac{1}{\sigma-1}}(w_1)^{-\frac{1}{\rho}}\,z_{11}^{*}. \tag{7}$$

Furthermore, (3) implies that z_{22}^{*} can be expressed as a function of z_{21}^{*}, and z_{11}^{*} can be expressed as a function of z_{12}^{*}. With a slight abuse of notation, we write these two functions as $z_{22}^{*}(z_{21}^{*})$ and $z_{11}^{*}(z_{12}^{*})$. Using these functions together with (6) leads to an expression that relates the productivity cut-off for exporting from 1 to 2, z_{12}^{*}, to the wage in country 1, w_1,

$$z_{12}^{*}=\tau_{12}\left(\frac{f_{12}}{f_{22}}\right)^{\frac{1}{\sigma-1}}(w_1)^{\frac{1}{\rho}}\,z_{22}^{*}\left(h_{21}\left(w_1,z_{11}^{*}\left(z_{12}^{*}\right)\right)\right). \tag{EXP}$$

Similarly, from (4) we can express M_i^e as a function of z_{12}^{*} and w_1, $M_i^e(w_1,z_{12}^{*})$, and then re-write the trade-balance condition as an equation in w_1 and z_{12}^{*},

$$\begin{aligned} & M_1^e(w_1,z_{12}^{*})f_{12}\left[J_1\left(z_{12}^{*}\right)+1-G_1(z_{12}^{*})\right] \\ = \quad & M_2^e(w_1,z_{12}^{*})f_{21}\left[J_2(h_{21}(w_1,z_{11}^{*}(z_{12}^{*})))+1-G_2(h_{21}(w_1,z_{11}^{*}(z_{12}^{*})))\right]. \end{aligned} \tag{TB}$$

This is also an equation in w_1 and z_{12}^{*}, which together with Condition EXP gives us a system of 2 equations in 2 unknowns. We can prove the following result:

Lemma 1 ***Condition EXP implies a positive relationship between*** w_1 ***and*** z_{12}^{*}***, while Condition TB implies a negative relationship between*** w_1 ***and*** z_{12}^{*}***.***

Proof. See the Appendix.

Conditions EXP and TB give us two curves, the "competitiveness curve" and the "trade balance curve," one sloping upwards and one downwards as shown in Figure 1, and their intersection gives the equilibrium values of w_1 and z_{12}^{*}.

2.4 Unilateral Trade Liberalization

We now can use the model to explore the effect of unilateral trade liberalization in country 1. In particular, we consider a reduction of inward variable and/or fixed trade barriers in country 1, τ_{21} and/or f_{21}. In this case, both conditions EXP and TB are affected the same way: for any fixed exporting productivity cut-off, wage must fall with a decline in barriers (see the Appendix for the proof). Therefore, both the competitiveness and trade balance curves move down. Unfortunately, in this case our graphical analysis does not provide us with the complete description of the new equilibrium: it is unclear what happens with the equilibrium cut-off z_{12}^{*} as it can potentially go up or down. Thus, one needs to go through the complicated mathematical derivations to get the answer. Nevertheless, knowing from our graphical analysis that w_1 falls with falling importing trade barriers significantly helps with the derivations, so we can prove that:

Proposition 1 *Welfare increases for a country that unilaterally reduces importing trade barriers.*

Proof. See the Appendix.

It is interesting to compare this result to that in Demidova (2008) for the setting with CES preferences and Melitz and Ottaviano (2008) for the setting with linear demand, where lowering trade barriers for foreign firms reduces welfare at Home. The reason for this result is that such liberalization in country 1 makes country 2 a better export base, which results in the additional entry of firms there. This entry intensifies competition, which results in less entry and lower welfare in country 1. Our model shows that this result no longer holds when there is no outside good pinning down the wage in both countries.

In the next Section we will show how the assumption that country 1 is a small economy used in Demidova and Rodríguez-Clare (2009) helps to significantly simplify the analysis.

3 Case of a Small Economy

Here we assume that country 1, which we now call "Home," can be treated as a small economy. Compared to Section 2, the small economy assumption requires two changes. First, we assume that foreign demand for a domestic variety is given by $Ap^{-\sigma}$. The term A includes both the national income and the price index in country 2, which we now call "Foreign." In line with the small economy assumption, A is not affected by changes at Home, i.e., A is exogenous in our small-country setting. Second, the measure M_2^e of monopolistically competitive firms in Foreign is exogenous. However, since $f_{21} > 0$, not all foreign firms sell at Home, so the measure of foreign varieties available at Home is endogenous.

In the Appendix we show that our small economy model can be obtained from the model of two large countries as a limit case, where the share of labor in Home, n, goes to zero. Formally, we show that if the two large countries are symmetric in everything except for size, and if the productivity distribution in both countries is Pareto, then in the limit as $n \to 0$ we obtain the three key assumptions of the small economy model, namely, that: (1) the domestic productivity cut-off for firms in Foreign is not affected by changes at Home; (2) the mass of firms in Foreign is not affected by changes at Home, and thus, the mass of available foreign varieties is fixed; and (3) the demand in Foreign for Home goods exported at the price p can be expressed as $Ap^{-\sigma}$, where A is a constant not affected by changes at Home.

3.1 Characterization of the Equilibrium

As before, productivity cut-offs z_{11}^* and z_{21}^* are determined by (1), but z_{22}^* is now taken as exogenous, while z_{12}^* is determined by

$$A\left(w_1\tau_{12}/\rho z_{12}^*\right)^{1-\sigma} = \sigma w_1 f_{12}. \tag{8}$$

In turn, the free entry, labor market clearing, and trade balance conditions at Home remain the same. To summarize, in the case of a small economy, there are 5 unknown variables in the equilibrium, M_1^e, z_{11}^*, z_{12}^*, z_{21}^*, and w_1, defined implicitly by 5 equilibrium equations.

3.2 Graphical Analysis

Next we will show how to reduce the system of 5 equilibrium conditions with 5 unknowns to 2 equations with 2 unknowns, w_1 and z_{12}^*. The first equation is obtained from (8),

$$z_{12}^* = \tau_{12} f_{12}^{1/(\sigma-1)} w_1^{1/\rho} (\sigma/A)^{1/(\sigma-1)} / \rho. \tag{EXP}$$

Note that this no longer depends on τ_{21} or f_{21}. The reason is that these conditions no longer affect country 2 (Foreign) if country 1 (Home) is small. This will simplify the comparative statics below.

The second equation is the trade balance condition and is the same as in the case of two large economies except that now M_2^e is now exogenous,

$$\begin{aligned} & M_1^e(w_1, z_{12}^*) f_{12} \left[J_1(z_{12}^*) + 1 - G_1(z_{12}^*) \right] \\ = \quad & M_2^e f_{21} \left[J_2(h_{21}(w_1, z_{11}^*(z_{12}^*))) + 1 - G_2(h_{21}(w_1, z_{11}^*(z_{12}^*))) \right]. \end{aligned} \tag{TB}$$

Conditions EXP and TB form a system of 2 equations in w_1 and z_{12}^*. Again, it can be shown that Condition EXP implies a positive relationship between w_1 and z_{12}^*, while Condition TB implies a negative relationship between w_1 and z_{12}^*. And with the same intuition as before, Conditions EXP and TB give us two curves, the "competitiveness curve" and the "trade balance curve," similar to those shown in Figure 1.

3.3 Unilateral Trade Liberalization

Now, consider a reduction of per-unit and/or fixed trade barriers for foreign exporters, τ_{21} and/or f_{21}. Unlike the case with a large Home economy, now only Condition TB is affected: for any fixed exporting productivity cut-off, the wage must fall with a decline in importing trade barriers at Home. Therefore, as shown in Figure 1(a), only the trade balance curve moves down, implying an unambiguous decline in the equilibrium levels of w_1 and z_{12}^*. As before, the decline in z_{12}^* implies an increase in z_{11}^* and, hence, an increase in the real wage in Home. The reason that the graphical analysis is now sufficient to establish the result is that the EXP curve does not depend on τ_{21} or f_{21}. In turn, this is because if Home is small then there is no feedback from changes in Home to the demand for Home goods in Foreign.

We can also use this analysis to explore the impact of reduction in the variable trade costs that Home faces to export goods to Foreign, i.e., a decline in τ_{12}. This leads to a shift up in the competitiveness curve shown in Figure 1(b), as a higher wage in Home is required to leave the export cut-off z_{12}^* unchanged when τ_{12} falls. But there is no shift in the trade balance curve, and hence, we immediately see that the decline in τ_{12} leads to an increase in Home's wage and a decline in the export cut-off z_{12}^*. The latter implies an increase in z_{11}^* and, hence, an increase in Home's real wage.

A decline in the fixed cost of exporting by Home firms in Foreign, f_{12}, is unfortunately not as simple. The reason is that now both the competitiveness and the trade balance curves shift with changes in f_{12}, where not only f_{12} enters the equations for both curves directly, but also affects the relationship between z_{11}^* and z_{12}^* implied by (3), i.e., the function $z_{11}^*(z_{12}^*)$ in Condition TB also depends on f_{12}, complicating the analysis.

4 Conclusion

The complexity of the Melitz model has led several researchers to adopt short-cuts in the analysis of trade liberalization in the presence of monopolistic competition, heterogenous firms, and fixed trade costs. Some have assumed that trade liberalization was symmetric in spite of the fact that liberalization was really asymmetric, often even unilateral. Some have instead added an outside good sector with zero trade costs as a way to fix relative wages, thereby ignoring general equilibrium forces that are important for the welfare analysis. In this paper we proposed an alternative approach that has a long history in the international trade literature, namely, that the country of interest is a small economy. This may miss important feedback effects when liberalization takes place in large economies, but for many cases of interest it seems like a reasonable approximation to reality. And the analytical benefits are significant – for example, the analysis of unilateral trade liberalization can be done with the help of a simple figure that helps to understand the key forces at play.

References

[1] Baldwin, R. E. and R. Forslid (2010). "Trade Liberalization with Heterogeneous Firms," *Review of Development Economics* 14(2):161-176.

[2] Baldwin, R. E. and T. Okubo (2009). "Tax Reform, Delocation, and Heterogeneous Firms," *Scandinavian Journal of Economics* 111(4):741-764.

[3] Chor, D. (2009). "Subsidies for FDI: Implications from a Model with Heterogenous Firms," *Journal of International Economics* 78:113-125.

[4] Demidova, S. (2008). "Productivity Improvements and Falling Trade Costs: Boon or Bane?", *International Economic Review* 49(4):1437-1462.

[5] Demidova, S. and A. Rodríguez-Clare (2009). "Trade Policy under Firm-Level Heterogeneity in a Small Economy," *Journal of International Economics* 78(1):100-112.

[6] Grossman, G. M., E. Helpman and A. Szeidl (2006). "Optimal Integration Strategies for the Multinational Firm," *Journal of International Economics* 70(1):216-238.

[7] Melitz, M. J. (2003). "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica* 71:1695-1725.

[8] Melitz, M. J. and G. I. P. Ottaviano (2008). "Market Size, Trade, and Productivity," *Review of Economic Studies* 75:295-316.

5 Appendix

5.1 Proof of Lemma 1

First, let us look at EXP: $z_{12}^* - \tau_{12}\left(\frac{f_{12}}{f_{22}}\right)^{\frac{1}{\sigma-1}}(w_1)^{\frac{1}{\rho}} z_{22}^* = 0$. We need to show that

$$\frac{dw_1}{dz_{12}^*} = -\frac{\partial LHS/\partial z_{12}^*}{\partial LHS/\partial w_1} > 0,$$

where $\partial LHS/\partial z_{12}^* = 1 - \tau_{12}\left(\frac{f_{12}}{f_{22}}\right)^{\frac{1}{\sigma-1}}(w_1)^{\frac{1}{\rho}} \frac{dz_{22}^*}{dz_{21}^*}\frac{dz_{21}^*}{dz_{11}^*}\frac{dz_{11}^*}{dz_{12}^*}$. By using (3) to derive dz_{ii}^*/dz_{ij}^*, and (7) to derive dz_{21}^*/dz_{11}^*, we get

$$\partial LHS/\partial z_{12}^* = 1 - \left(\frac{f_{12}f_{21}}{f_{11}f_{22}}\tau_{12}\tau_{21}\left(\frac{f_{12}f_{21}}{f_{11}f_{22}}\right)^{\frac{1}{\sigma-1}}\right)^2 \frac{J_1'(z_{12}^*)\,J_2'(z_{21}^*)}{J_1'(z_{11}^*)\,J_2'(z_{22}^*)},$$

where $J_i'(a) = \frac{1-\sigma}{a}\int_a^\infty \left(\frac{\varphi}{a}\right)^{\sigma-1} dG_i(\varphi)$. Using EXP and (7), we get

$$\partial LHS/\partial z_{12}^* = 1 - (\tau_{12}\tau_{21})^{2(1-\sigma)} \frac{\int_{z_{12}^*}^\infty \varphi^{\sigma-1} dG_1(\varphi)\int_{z_{21}^*}^\infty \varphi^{\sigma-1} dG_2(\varphi)}{\int_{z_{11}^*}^\infty \varphi^{\sigma-1} dG_1(\varphi)\int_{z_{22}^*}^\infty \varphi^{\sigma-1} dG_2(\varphi)} > 0,$$

since $\tau_{12}\tau_{21} > 1$, $z_{11}^* < z_{12}^*$, and $z_{22}^* < z_{21}^*$. Next, note that

$$\partial LHS/\partial w_1 = -\frac{z_{12}^*}{\rho w_1} - \tau_{12}\left(\frac{f_{12}}{f_{22}}\right)^{\frac{1}{\sigma-1}}(w_1)^{\frac{1}{\rho}}\frac{dz_{22}^*}{dz_{21}^*}\frac{dz_{21}^*}{dw_1} < 0,$$

since from (3) and (7), $dz_{22}^*/dz_{21}^* < 0$ and $dz_{21}^*/dw_1 = -z_{21}^*/\rho w_1$. Hence, from EXP, $dw_1/dz_{12}^* > 0$.

Finally, let us look at TB. Denote $\left(f_{ii}\int_{z_{ii}^*}^\infty (\varphi/z_{ii}^*)^{\sigma-1} dG_i(\varphi)\right) / \left(f_{ij}\int_{z_{ij}^*}^\infty \left(\varphi/z_{ij}^*\right)^{\sigma-1} dG_i(\varphi)\right)$ by ψ_i. Given w_1 from (7), and using (4), TB can be rewritten as

$$L_2\left(\frac{f_{22}\left[J_2(z_{22}^*) + 1 - G_2(z_{22}^*)\right]}{f_{21}\left[J_2(z_{21}^*) + 1 - G_2(z_{21}^*)\right]} + 1\right)^{-1} = \frac{w_1 L_1}{\sigma}\left(\frac{f_{11}\left[J_1(z_{11}^*) + 1 - G_1(z_{11}^*)\right]}{f_{12}\left[J_1(z_{12}^*) + 1 - G_1(z_{12}^*)\right]} + 1\right)^{-1},$$

or

$$\left(\tau_{21}(f_{21}/f_{11})^{\frac{1}{\sigma-1}}\right)^{\rho}(z_{21}^*)^{-\rho}(\psi_2 + 1) = (z_{11}^*)^{-\rho}(\psi_1 + 1). \tag{9}$$

The RHS of (9) can be written as a function of z_{12}^*. From (3), it rises with z_{12}^*. The LHS of (9) can be written as a function of z_{21}^*. Again from (3), the LHS of (9) rises with z_{21}^*. Thus, from TB it follows that if z_{12}^* rises, then z_{21}^* must rise as well. Moreover, from (3), z_{11}^* must fall with rising z_{12}^*. Using these conclusions together in (7), we proved that from TB, w_1 falls with z_{12}^*.

5.2 Proof of Proposition 1

Shift in the curves. First, let us show that for any given z_{12}^*, a decrease in τ_{21} and/or f_{21} shifts down the competitiveness curve. To see this, note that if z_{12}^* is fixed, then from (3), z_{11}^* is fixed as well. But since from EXP and (7), $z_{12}^* z_{21}^* = \tau_{12}\tau_{21}(f_{12}f_{21}/f_{11}f_{22})^{\frac{1}{\sigma-1}} z_{11}^* z_{22}^*$, z_{22}^* must rise and z_{21}^*

must fall (from (3) they move in the opposite directions). Hence, from EXP, w_1 falls for any fixed z_{12}^*.

Now we need to show that for any fixed w_1, a decrease in τ_{21} and/or f_{21} shifts the trade balance curve to the left, i.e., z_{12}^* falls for any given w_1. First, note that as τ_{21} (and/or f_{21}) falls, then z_{21}^* must fall as well. To see this, for a fixed w_1, we can rewrite (7) and (9) as

$$\begin{aligned} \tau_{21}\left(\frac{f_{21}}{f_{11}}\right)^{\frac{1}{\sigma-1}}\frac{z_{11}^*}{z_{21}^*} &= (w_1)^{\frac{1}{\rho}} \equiv Const_1, & (10)\\ (\psi_1+1) &= w_1(\psi_2+1) \equiv Const_2 * (\psi_2+1). & (11)\end{aligned}$$

Assume that z_{21}^* rises. Then from (10), z_{11}^* must rise as well. However, if z_{21}^* rises, then from (3), z_{22}^* falls, resulting in falling ψ_2 and (from (11)) falling ψ_1, which from (3) implies that z_{11}^* falls, leading to contradiction. Hence, z_{21}^* falls with a fall in τ_{21} (and/or f_{21}).

Next, in the case of falling τ_{21}, a fall in z_{21}^* raises z_{22}^* and decreases ψ_2, so that ψ_1 falls as well, implying a fall in z_{12}^*, which we wanted to prove. However, in the case of falling f_{21} we cannot use the same logic, since f_{21} enters the free entry condition (3) for country 2. Let us assume that z_{12}^* rises. Then from (3), z_{11}^* falls and, in turn, ψ_1 rises so that from (11), ψ_2 must rise as well. But

$$\psi_2 = \frac{f_{22}\int_{z_{22}^*}^{\infty}(\varphi/z_{22}^*)^{\sigma-1}dG_2(\varphi)}{\left((f_{21})^{\frac{1}{\sigma-1}}/z_{21}^*\right)^{\sigma-1}\int_{z_{21}^*}^{\infty}\varphi^{\sigma-1}dG_2(\varphi)}, \qquad (12)$$

where, as we proved before, z_{21}^* falls, and from (10), $(f_{21})^{\frac{1}{\sigma-1}}/z_{21}^*$ must rise with falling z_{11}^* and τ_{21}. Hence, the denominator in (12) rises, so for ψ_2 to rise, z_{22}^* must fall. Then from (3), $f_{21}J_2(z_{21}^*)$ should fall as well. However, since z_{21}^* falls with falling f_{21},

$$f_{21}J_2(z_{21}^*) = \left((f_{21})^{\frac{1}{\sigma-1}}/z_{21}^*\right)^{\sigma-1}\left[\int_{z_{21}^*}^{\infty}\varphi^{\sigma-1}dG_2(\varphi) - (z_{21}^*)^{\sigma-1}(1-G(z_{21}^*))\right]$$

must rise, not fall, which leads to contradiction. Thus, we proved that for any given w_1, z_{12}^* falls with a fall in f_{21}.

Welfare change. We know from Figure 1 that if both curves shift down, w_1 falls with a fall in τ_{21} and/or f_{21}. Can z_{12}^* rise as a result? Assume that yes. Then from (3), z_{11}^* falls. This means that in (9) rewritten as $w_1(\psi_2+1) = (\psi_1+1)$, ψ_1 rises. Hence, the LHS of (9) must rise as well. Then in the case of a fall in τ_{21} this means that from (3), z_{21}^* must rise and z_{22}^* must fall. But, from (6) z_{22}^* must rise, which results in contradiction. Thus, in the case of falling τ_{21} z_{12}^* falls. The case of falling f_{21} is more complicated since f_{21} is a part of ψ_2. To deal with it, let us rewrite ψ_2 as

$$\psi_2 = \frac{f_{22}(z_{22}^*)^{1-\sigma}\int_{z_{22}^*}^{\infty}\varphi^{\sigma-1}dG_2(\varphi)}{f_{21}(z_{21}^*)^{\sigma-1}\int_{z_{21}^*}^{\infty}\varphi^{\sigma-1}dG_2(\varphi)} = \frac{(\tau_{12}\tau_{21})^{\sigma-1}f_{12}(z_{12}^*)^{1-\sigma}\int_{z_{22}^*}^{\infty}\varphi^{\sigma-1}dG_2(\varphi)}{f_{11}(z_{11}^*)^{1-\sigma}\int_{z_{21}^*}^{\infty}\varphi^{\sigma-1}dG_2(\varphi)},$$

where the last equality follows from (6) and (7). Then since w_1 and z_{11}^* fall, while z_{22}^* and z_{12}^* rise, for the LHS of (9) to rise, z_{21}^* must rise. However, if f_{21} falls, while z_{22}^* and z_{21}^* rise, then the LHS of (3) for the Foreign country falls, while the RHS remains constant, which leads to contradiction. Hence, as in the case of falling τ_{21}, z_{12}^* falls with a fall in f_{21}. Therefore, from (3), z_{11}^* rises, raising welfare at Home.

5.3 Justification of Small Economy Assumptions

Here we will show that the assumptions we use to treat Home as a small economy can be obtained from the model of two large countries, Home and Foreign, with Home becoming small relative to the Foreign one (the "limit" case). In particular, if two countries are endowed with n and $(1-n)$ shares of the world's labor, L,

$$L_1 = nL, \quad L_2 = (1-n)\,L, \quad n \in [0,1]\,,$$

then the "limit" case we want to explore is the one when $n \to 0$. The assumptions we want to explain by the "limit" case are: (1) The domestic productivity cutoff for foreign firms (and, therefore, the productivity distribution of the active firms there) is not affected by changes at Home; (2) The mass of firms in the Foreign country is not affected by changes at Home, and thus, the mass of available foreign varieties is fixed; and (3) The foreign demand for Home goods exported at the price p can be expressed as $Ap^{-\sigma}$, where A is a constant not affected by changes at Home.

To simplify our analysis, we assume that 2 countries are symmetric in everything except for their sizes, i.e., $f_{11} = f_{22} = f$, $f_{12} = f_{21} = f_x$, $F_1 = F_2 = F_e$, $\tau_{12} = \tau_{21} = \tau$. Also, we assume that the productivity distribution in both countries is now specified as Pareto: $G(\phi) = 1 - \left(\frac{b}{\phi}\right)^{\beta}$ for $\phi \geq b$. Then, the free entry condition in country i can be written as

$$(\theta - 1)\, b^{\beta} \left[f\,(z_{ii}^*)^{-\beta} + f_x \left(z_{ij}^*\right)^{-\beta} \right] = F_e, \tag{FE}$$

where $\theta = \beta / \left(\beta - (\sigma - 1)\right)$. Moreover, from (6) and (7),

$$z_{ij}^* = \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \left(\frac{w_i}{w_j}\right)^{\frac{\sigma}{\sigma-1}} z_{jj}^* \equiv B \left(\frac{w_i}{w_j}\right)^{\frac{\sigma}{\sigma-1}} z_{jj}^*, \quad \text{where } B \equiv \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} > 1.$$

Note that by using FE in the definition of M_i^e, we get

$$M_i^e = \frac{(\theta-1)\, b^{\beta}}{\sigma F_e} L_i.$$

Hence, if we denote $\frac{w_1}{w_2}$ by w, then we get the new TB condition:

$$\frac{n}{1-n} = w^{2\beta\frac{\sigma}{\sigma-1}-1} \left[\frac{z_{11}^*}{z_{22}^*}\right]^{-\beta}. \tag{TB}$$

To summarize, for given n, the equilibrium in the model with 2 countries can be described by 2 free entry and 1 trade balance conditions and 3 unknown variables, z_{11}^*, z_{22}^*, and w.

What happens in the model described above when $n \to 0$? Solving FE for z_{11}^* and z_{22}^* gives

$$\left[\frac{z_{11}^*}{z_{22}^*}\right]^{-\beta} = \frac{1 - \frac{f_x}{f} B^{-\beta} w^{-\beta\frac{\sigma}{\sigma-1}}}{1 - \frac{f_x}{f} B^{-\beta} w^{\beta\frac{\sigma}{\sigma-1}}},$$

so that the TB condition can be rewritten as

$$\frac{n}{1-n} = w^{2\beta\frac{\sigma}{\sigma-1}-1} \frac{1 - \frac{f_x}{f} B^{-\beta} w^{-\beta\frac{\sigma}{\sigma-1}}}{1 - \frac{f_x}{f} B^{-\beta} w^{\beta\frac{\sigma}{\sigma-1}}}. \tag{13}$$

As $n \to 0$, the LHS of (13) goes to 0. Moreover, the RHS of (13) rises with w (here we use the fact that $\frac{f_x}{f}B^{-\beta} < 1$). Hence, as n falls, w falls as well, and when $n \to 0$, the RHS of (13) goes to 0. Note that if $n < 1/2$, then $w < 1$. (If $w > 1$, then from FE, $z_{11}^* < z_{22}^*$. But then in (13), the LHS<1, while the RHS>1, resulting in contradiction.). Thus, the denominator in the RHS of (13) is always positive and bigger than $1 - \frac{f_x}{f}B^{-\beta}$. Hence, as $n \to 0$, we must have $w^{2\beta\frac{\sigma}{\sigma-1}-1}\left(1 - \frac{f_x}{f}B^{-\beta}w^{-\beta\frac{\sigma}{\sigma-1}}\right) \to 0$. Can w be below $\left[\frac{f_x}{f}B^{-\beta}\right]^{\frac{\sigma-1}{\beta\sigma}}$ for some $n \in (0, 1/2)$? The answer is no, since in this case the RHS of (13) would become negative, while $n/(1-n) > 0$. Thus, as $n \to 0$, then w falls to $\left[\frac{f_x}{f}B^{-\beta}\right]^{\frac{\sigma-1}{\beta\sigma}}$. Moreover, from FE, if n falls, then z_{22}^* falls and z_{11}^* rises.

Note that due to the Pareto distribution assumption, z_{22}^* cannot fall below b, the minimum value for ϕ, but from the solution of FE, it seems that $z_{22}^* \to 0$ as $n \to 0$. How to explain this? The reason is that as n continues to fall, z_{22}^* reaches its minimum so that all foreign firms survive. As n continues to fall, z_{22}^* remains at level b, and the zero profit condition for country 2 is violated, so that FE is no longer true for country 2.[4] This also means that we proved assumption (1): productivity cutoff z_{22}^* is not affected by changes at Home, when n is small enough.

Now let us derive the new FE conditions for n small enough so that $z_{22}^* = b$ and $\pi_{22}(z_{22}^*) > 0$. While for Home we have the same FE condition as before, for the Foreign country,

$$\frac{1}{\sigma}L_2P_2^{\sigma-1}\rho^{\sigma-1}\theta b^{\sigma-1} - f + f_x(\theta-1)b^\beta(z_{21}^*)^{-\beta} = F_e,$$

which from the zero profit condition for exporters from Home can be rewritten as

$$wf_x\left(\frac{w\tau}{z_{12}^*}\right)^{\sigma-1}\theta b^{\sigma-1} - f + f_x(\theta-1)b^\beta(z_{21}^*)^{-\beta} = F_e. \tag{FE}$$

By using the new FE conditions for small enough n, we get

$$M_1^e = \frac{(\theta-1)b^\beta nL}{\sigma F_e}, \quad M_2^e = \frac{(1-n)L}{\sigma\left(F_e + f + b^\beta f_x(z_{21}^*)^{-\beta}\right)},$$

which allows us to rewrite the TB condition as

$$\frac{n}{1-n} = \frac{F_e(z_{12}^*/z_{21}^*)^\beta}{(\theta-1)b^\beta w\left(F_e + f + b^\beta f_x(z_{21}^*)^{-\beta}\right)}.$$

As $n \to 0$, the LHS falls to 0 as well. Since the minimum value for $F_e + f + b^\beta f_x(z_{21}^*)^{-\beta}$ cannot be smaller than $F_e + f$, $(z_{12}^*/z_{21}^*)^\beta/w \to 0$ as $n \to 0$. Using this property in the new FE condition for country 2, which we can rewrite as

$$f_x\left(\frac{w\tau}{z_{12}^*}\right)^{\sigma-1}\theta b^{\sigma-1}(z_{12}^*)^\beta + f_x(\theta-1)b^\beta\left[(z_{12}^*/z_{21}^*)^\beta/w\right] = (F_e+f)\frac{(z_{12}^*)^\beta}{w},$$

means that we can ignore the second term in the LHS above, i.e., for small enough n,

$$f_x\left(\frac{w\tau}{z_{12}^*}\right)^{\sigma-1}\theta b^{\sigma-1}(z_{12}^*)^\beta \sim (F_e+f)\frac{(z_{12}^*)^\beta}{w}, \quad \text{or} \quad w^\sigma(z_{12}^*)^{1-\sigma} \sim const.$$

[4] Note that this logic also applies to the other types of the productivity distributions.

However, from the zero profit condition for exporters from Home, $R_2 P_2^{\sigma-1} \propto w^{\sigma} \left(z_{12}^*\right)^{1-\sigma}$. Hence, we proved assumption (3): at some low level of n, we can treat $R_2 P_2^{\sigma-1}$ as a constant, i.e., the foreign demand for Home goods exported at the price p can be expressed as $Ap^{-\sigma}$. This also means that since for small n, $P_2^{1-\sigma} = M_2^e \theta \rho^{\sigma-1} b^{\beta} + M_1^e \theta b^{\beta} \left(\rho/\tau w\right)^{\sigma-1} \left(z_{12}^*\right)^{-\beta+(\sigma-1)} \sim M_2^e \theta \rho^{\sigma-1} b^{\beta}$ (as L_1 is very small) and $R_2 \sim L$, then treating $R_2 P_2^{\sigma-1}$ as a constant implies treating M_2^e as a constant, i.e., we proved assumption (2).