Econ 559
Human Capital and Growth
Based on Acemoglu, Chapter 10

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General model of individual consumption and human capital accumulation is summarized by:

$$\int_0^T e^{-(\rho+v)t} u(c(t))dt$$

with budget constraint with perfect capital markets,

$$\int_0^T e^{-rt} c(t)dt \leq \int_0^T e^{-rt} W(t)dt$$

and

$$\begin{aligned} W(t) &= w(t)\,[1 - s(t)]\,h(t) \\ \dot{h}(t) &= G(t, h(t), s(t)),\ s(t) \in \mathbf{S}(t) \subset [0,1] \end{aligned}$$

The **separation theorem** shows that under these conditions (include no leisure in the utility function) we can separate consumption and human capital accumulation decisions. In this case, human capital decisions are made to maximize the present value of lifetime earnings.

Assume now that $T = \infty$ with a flow rate of death $v > 0$ (perpetual youth with finite expected life), and assume that for an individual born at time $t = 0$ schooling entails $s(t) = 1$ for $t \in [0, S]$ with $s(t) = 0$ for $t > S$.

Assume also that $h(S) = \eta(S)$, with $\eta' > 0$ and $\eta'' < 0$, with $\dot{h}(t) = g_h h(t)$ for $t \geq S$ for $g_h \geq 0$.

Wages $w(t)$ (per unit of human capital) grow exponentially at rate g_w and

$$r + v > g_h + g_w$$

Applying the separation theorem, S is chosen to maximize

$$\begin{aligned} & \int_S^\infty e^{-rt} w(t) h(t) dt \\ = \; & h(S) w(S) \int_S^\infty e^{-(r+v)t} e^{g_w (t-S)} e^{g_h (t-S)} dt \\ = \; & \frac{\eta(S) w(0) e^{g_h S} e^{-(r+v-g_w-g_h)S}}{r+v-g_w-g_h} \\ = \; & \frac{\eta(S) w(0) e^{-(r+v-g_w)S}}{r+v-g_w-g_h} \end{aligned}$$

Taking logs, the FOC for S is

$$\frac{\eta'(S^*)}{\eta(S^*)} = r + v - g_w$$

With $\eta'(S)/\eta(S)$ decreasing, a solution S^* characterizes an optimal schooling decision that is decreasing in r and v and increasing in g_w.

Integrating $\frac{\eta'(S^*)}{\eta(S^*)} = r + v - g_w$ yields

$$\log \eta(S^*) = c \ + (r + v - g_w)S^*$$

Using

$$W(S^*, t) = w(0)e^{g_w t}e^{g_h(t-S^*)}\eta(S^*)$$

then

$$\log W(S^*, t) = c + (r + v - g_w)S^* + g_w t + g_h(t - S^*)$$

This can be implemented empirically as

$$\log W_j = c + \gamma_s S_j + \gamma_e \cdot experience + \varepsilon_j$$

assuming that ε_j is independent of S_j.

Note that with $v \approx g_w \simeq 2\%$, then γ_s should be close to r. We can think of a rough measure of r from

$$r = \frac{rP_k K / P_y Y}{P_k K / P_y Y} = \frac{\alpha(K)}{P_k K / P_y Y}$$

which is between 7% and 9% (although higher in rich than in poor countries... see Caselli and Feyrer's "The Marginal Product of Capital" in the QJE 2007). This is consistent with $\gamma_s \in [0.06, 0.1]$.

How do we get variation in S across people?

One possibility is differences in ability as in $\eta_j(S) = z_j \eta(S)$, but since this would increase both the cost and benefit of schooling, there would be no effect on S, which would be common across people. The effect on the wage would be captured by ε_j.

Another possibility is that ability affects the way in which schooling determines human capital, as in

$$\eta_j(S) = \exp\left[b_j S - (k/2)S^2\right]$$

Then $\eta_j'(S)/\eta_j(S) = b_j - kS$, and a higher b_j leads to a higher $S_j^* = \left(b_j - \gamma\right)/k$, where $\gamma \equiv r + v - g_w$.

At the optimal level of schooling, individual $j's$ marginal return to schooling is the same for all j, given by γ. Now have

$$\frac{d \log \eta_j}{dS_j} = b_j - kS_j^* = b_j - k\left(b_j - \gamma\right)/k = \gamma$$

so

$$\log \eta_j = \alpha + \gamma S_j^* + \varepsilon_j$$

The problem is that now ε_j and S_j are correlated. In particular, given

$$\log \eta_j = b_j S_j^* - (k/2)\left(S_j^*\right)^2 = \alpha + \gamma S_j^* + \varepsilon_j$$

then (plugging in for $S_j^* = \left(b_j - \gamma\right)/k$ and simplifying)

$$\varepsilon_j = -\alpha + (k/2)\left(S_j^*\right)^2$$

There are two problems with running the Mincer OLS regression to estimate γ.

First, the true effect of S on log earnings is not constant but decreasing in S, because $d \log \eta_j(S)/dS = b_j - kS$.

Second, the correlation between ε_j and S_j generates a convex relationship between log earnings and schooling.

In "The Causal Effect of Education on Earnings" in the Handbook of Labor Economics (Volume 3A), David Card argues that success of the linear Mincer specification is likely to come from the fact that these two effects cancel out.

The Ben-Porath model

Allow $s(t) \in [0, 1]$ for all $t \geq 0$, with perpertual youth ($v > 0$), a constant wage w, a constant interest rate r, and

$$\dot{h}(t) = \phi(s(t)h(t)) - \delta_h h(t)$$

with some $h(0) > 0$ and $\phi' > 0$, $\phi'' < 0$, $\lim_{x \to 0} \phi'(x) = \infty$.

Using $x(t) \equiv s(t)h(t)$ then earnings are $(1 - s(t))\, h(t) = h(t) - x(t)$, so individual's problem is to choose $x(t)$ to

$$\max \int_0^\infty e^{-(r+v)t}(h(t) - x(t))dt$$

subject to $\dot{h}(t) = \phi(x(t)) - \delta_h h(t)$.

Setting up the current-value Hamiltonian and following the usual steps we find

$$1 = \mu(t)\phi'(x(t))$$

and

$$\frac{\dot{\mu}(t)}{\mu(t)} = r + v + \delta_h - \phi'(x(t))$$

with TC given by

$$\lim_{t \to \infty} e^{-(r+v)t}\mu(t)h(t) = 0$$

The steady state is given by

$$x^* = \phi'^{-1}(r + v + \delta_h)$$

with

$$h^* = \phi(x^*)/\delta_h$$

Can show that x^* and h^* are decreasing in $r + v + \delta_h$ (since $\phi'' < 0$).

Differentiating $1 = \mu(t)\phi'(x(t))$ w.r.t. t and using $\varepsilon_{\phi'}(x) \equiv -x\phi''(x)/\phi'(x) > 0$ implies

$$\frac{\dot{\mu}(t)}{\mu(t)} = \varepsilon_{\phi'}(x)\frac{\dot{x}(t)}{x(t)}$$

and hence

$$\frac{\dot{x}(t)}{x(t)} = \frac{1}{\varepsilon_{\phi'}(x)}\left[r + v + \delta_h - \phi'(x(t))\right]$$

with $\dot{h}(t) = \phi(s(t)h(t)) - \delta_h h(t)$.

The phase diagram reveals that (h^*, x^*) is globally saddle-path stable, with convergence taking place with $x(t) = x^*$. Note: this is OK as long as $x^* < h(0)$, otherwise need to take the constraint $x(t) \in [0, h(t)]$ explicitly in the problem. A sufficient condition for $x^* < h(0)$ is $\lim_{x \to h(0)} \phi'(x) = 0$.

The nice thing about this solution is that with $x(t) = x^*$ and $h(t)$ increasing from $h(0)$ towards h^*, then $s(t)$ will be very high at the beginning and decline with time.