Econ 559
Economics of Innovation
Based on Acemoglu, Chapter 12

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Main points of chapter 12:

- Types of technological change: process and product innovation

- Cost reductions = quality improvements

- Non-rivalry of ideas: $F(K, L, A)$ is CRS in K and L and IRS to K, L and A

- The presence of IRS leads to market-size effects

- Technological change arises both because of scientific progress and profit-driven innovation

- No innovation in pure competition - caveats

- Monopoly versus limit pricing: drastic innovation defined as large enough that it allows firm to charge monopoly price and still capture the whole market – $p^M \leq \psi$, otherwise capture the whole market but with limit pricing – $p_1 = \psi$

- The appropriability effect: the social value of an innovation under competitive price ($p = \psi/\lambda$) is higher than the private value (for a monopolist)

- The replacement effect: a potential entrant has stronger incentives to undertake an innovation than does an incumbent monopolist.

  – The private value of an innovation (for a firm in the "fringe") can be higher than the social value

The Dixit-Stiglitz Model

Preferences of representative household are

$$U(c_1, ..., c_N, y) = u(C, y)$$

with

$$C = \left(\sum c_i^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}$$

and $\varepsilon > 1$.

Function $u(C, y)$ is strictly increasing, differentiable and jointly strictly concave.

Love of variety. Consider $c_i = X/N$ for all i, so that X is the total number of units of differentiated commodities. Then

$$C = \left(\sum (X/N)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} = XN^{\frac{1}{\varepsilon-1}}$$

which is increasing in N given $\varepsilon > 1$.

The consumer's problem is to maximize utility given budget constraint $\sum p_i c_i + y \leq m$. FOC is

$$(c_i/c_l)^{-1/\varepsilon} = p_i/p_l$$

Let

$$P \equiv \left(\sum p_i^{1-\varepsilon}\right)^{1/(1-\varepsilon)}$$

This is the ideal price index: it satisfies $C = E/P$, where E is expenditure on differentiated goods.

Note also that

$$\begin{aligned}
\frac{c_i}{c_l} &= \frac{p_i^{-\varepsilon}}{p_l^{-\varepsilon}} \\
p_i c_i p_l^{-\varepsilon} &= p_i^{1-\varepsilon} c_l \\
p_l^{-\varepsilon} \sum p_i c_i &= c_l \sum p_i^{1-\varepsilon} \\
c_l &= E p_l^{-\varepsilon} / P^{1-\varepsilon}
\end{aligned}$$

Checking:

$$\begin{aligned}
c_l^{(\varepsilon-1)/\varepsilon} &= E^{(\varepsilon-1)/\varepsilon} p_l^{1-\varepsilon} P^{(\varepsilon-1)^2/\varepsilon} \\
C^{(\varepsilon-1)/\varepsilon} &= E^{(\varepsilon-1)/\varepsilon} P^{1-\varepsilon} P^{(\varepsilon-1)^2/\varepsilon} \\
C &= E P^{-\varepsilon} P^{\varepsilon-1} = E/P
\end{aligned}$$

The result $c_l = E p_l^{-\varepsilon} / P^{1-\varepsilon}$ can be restated as

$$c_l = (E/P)(p_l/P)^{-\varepsilon} = C(p_l/P)^{-\varepsilon}$$

or (in terms of expenditure)

$$p_l c_l = E\,(p_l/P)^{1-\varepsilon}$$

At the same time, consumer chooses C and y to maximize $U(C, y)$ s.t. $PC + y \leq m$. This yields $y = g(P, m)$ and $C(m, P) = [m - g(P, m)]\,/P$.

Monopolists' problem is $\max_{p_l \geq 0} CP^{\varepsilon} p_l^{-\varepsilon} (p_l - \psi)$. Since P depends on P_l then this is a potentially complicated problem. But note that

$$\begin{aligned} \partial P/\partial p_l &= \frac{1}{1-\varepsilon} \left(\sum p_i^{1-\varepsilon} \right)^{1/(1-\varepsilon)-1} (1-\varepsilon) p_l^{-\varepsilon} \\ &= P^{\varepsilon} p_l^{-\varepsilon} = (p_l/P)^{-\varepsilon} = (p_l c_l/E)^{\varepsilon/(\varepsilon-1)} \end{aligned}$$

If the share of spending on l is small (large N) then we can disregard this effect. This is the standard trick. It is exactly right with a continuum of varieties.

In this case the monopolists' problem is standard and we get

$$p_l = p = \frac{\varepsilon}{\varepsilon - 1} \psi = \frac{1}{1 - 1/\varepsilon} \psi \text{ for all } l$$

This implies that $P = pN^{1/(1-\varepsilon)}$, sales are E/N, and profits are

$$\pi = (E/N)(1/p)(p - \psi) = R/\varepsilon$$

If $\varepsilon = 5$ then this says that profits are $1/5$ of total revenue, while the mark-up is $1/(1 - 1/5) = 5/4 = 1.25$.

Profits can also be written as

$$\pi = N^{-\varepsilon/(\varepsilon-1)}C\left(\frac{\psi}{\varepsilon-1}\right)$$

This is decreasing in ε and N (given E), and increasing in E. But C is a function of P and P is a function of N... so effect of P on π can be positive depending on the form of $g()$ - this is an "aggregate demand externality."

With a continuum of varieties, assuming an entry cost of μ, the equilibrium entails $\pi(N) = \mu$.

What about efficiency? Is N^* efficient or is there too much entry?

Before Dixit-Stiglitz people thought that there was too much entry based on the fact that MR = MC with zero profits necessarily implies that the AC curve is downward sloping. But this missed the fact that entry itself is valuable - there is a trade-off between AC and variety.

Does the equilibrium strike the right balance? In fact, one can show that if $y = g(P, m) = 0$ then the equilibrium generates the optimal variety.

Do it yourself or check my notes titled "Optimum vs Equilibrium Variety in the Dixit-Stiglitz Model" under course 507 in my website.