Electoral Competition with Uncertainty Averse Parties

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Abstract

The nonexistence of equilibria in models of electoral competition involving multiple issues is one of the more puzzling results in political economics. In this paper we relax the standard assumption that parties act as expected utility maximizers. We show that equilibria often exist when parties with limited knowledge about the electorate are modelled as uncertainty averse. What is more, these equilibria can be characterized as a straightforward generalization of the classical median voter result.

Keywords: Uncertainty Aversion, Mutliple Priors, Median Voter, Electoral Competition over many Issues. **JEL Classification Numbers:** D72, D81.

1 Introduction

The famous median voter theorem states that in a platform positioning game with a unidimensional issue space played by two office motivated parties and a set of voters with single peaked preferences, both parties will announce the policy preferred by the median voter in equilibrium (Downs 1957). The assumption that the political spectrum is unidimensional is crucial. Most results for multidimensional games of this sort show that equilibria exist only if the distribution of voters satisfies very strong conditions (Plott (1967), Davis, Hinich and de Groot (1972) and Grandmont (1978), McKelvey (1979)). This is problematic since

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governments in real existing democracies do decide on many different issues and the assumption that all these issues can be aligned perfectly on a unidimensional spectrum (say left to right) is quite strong. Noting the discord between life (multidimensional stable politics) and theory (either uni-dimensional stable poltics or multidimensional chaos) Ordeshook (1986) conlcudes his discussion of the non-existence issues in games of platform positioning in his book "Game Theory and Political Theory" as follows: "We should keep in mind, of course, our examples do not match reality. Candidates are uncertain about voter's exact preferences, and voters are only imperfectly informed about candidates' strategies... ."

We follow this suggestion by Ordeshook (1986) by assuming that parties or candiates are uncertain about the voter's preferences. In justification of this assumption of subjective uncertainty observe that parties do indeed face choices with ambiguous consequences. Electoral outcomes are generally hard to predict: elections are usually held in ever changing environments, involving new issues, turnover of party elites, and fluctuations amongst the electorate. We follow the decision theoretic literature on ambiguity aversion by assuming that parties prefer strategies with less ambiguous outcomes. Such a bias against ambiguity is manifest in many studies on individual choice behavior. When facing uncertain situations the choices of individuals tend to violate the independence axiom, a preference for options involving less uncertainty manifests itself in many experiments (see Camerer and Weber (1992) for a review). Party platforms are, of course, not set by single individuals but by large groups. Even if we assume that every party member is uncertainty neutral, uncertainty averse behavior by the party as a whole arises if different party members or factions hold different beliefs about the electorate and if parties change their platforms only if such a change is viewed as beneficial by all party members. Roemer (1999) and Levy (2004) model the decision making of parties in a similar manner, the main difference between their models and the present one being that different party members pursue different *goals* in their papers whereas party members hold different *beliefs* in the present model. We also mention in passing that there is some empirical evidence, provided by Keller, Sarin and Sounderpandian (2002) that groups act more uncertainty averse than individuals.

The incorporation of subjective uncertainty about the electorate together with the assumption of ambiguity aversion of parties has a major payoff for the multi-dimensional Downsian model of platform positioning. In many cases the problem of equilibrium nonexistence can be mitigated. What is more, the equilibrium prediction of the model with uncerainty averse parties extends equilibrium prediction of the classic Downs model in a natural manner to the environment with multiple issues: each party will with respect to every plank announce a policy that this the most preferred policy by some (suitably defined) meadian voter. It is important to note that beliefs on the preferences of the electorate do not matter in the Downsian paradigm if both parties announce the same platform; in this case all voters are indifferent and both parties receive an expected vote share of one half - no matter the parties beliefs on the preferences of the electorate. So if parties are uncertain about voter preferences, this uncertainty matters only as long as both parties announce different platforms. This feature of the Downsian model is essential for our argument: our assumption of uncertainty aversion introduces a bias towards adopting the platform of the opponent. If this bias is strong enough to outweigh the desire to deviate from the platform of the other party, equilibrium existence is obtained. The characterization of the equilibrium outcome as the median voter's most preferred platform - issue by issue - is owed to the assumption that preferences of voters are not only single peaked but also separable.

There is a range of different proposals on how to represent the preferences of uncertainty averse agents, notably Bewley (2002), Gilboa and Schmeidler (1989), Schmeidler (1989), Ghirardato et al. (2003) and Maccheroni et al. (2006). Since the main intuition for the existence result presented in this article is rooted in the Ellsberg paradox and since all of these approaches are able to explain the Ellsberg paradox the main arguments of this paper could be based on any of these approaches. Gilboa and Schmeidler's (1989) approach was chosen, since their model allows for tractable results and since it has alreadyfound some application outside the narrow confines of decision theory. For a survey of applications of the decision theory of ambiguity aversion to various contexts of economics see Mukerji and Tallon (2004). In political economy, Ghirardato and Katz (2002) use uncertainty aversion to explain the empirical puzzle that voters abstain from elections, even when voting is costless, i.e. when the voter is already in the booth because of a different election. Ashworth (2007) usess a similar framwork to explain parties decisions to target some groups of voters to affect turnout and to target other groups to switch their allegiance.Berliant and Konishi's (2005) explanation of the fact that parties avoid to discuss some issues in electoral campaigns is most related to the present paper, as the driving force behind their argument is the assumption that parties are uncertainty averse.

Other proposals to remedy the problem of equilibrium non-existence in platform positioning games with multidimensional issue spaces include Besley and Coate (1997), Roemer (1999) and Duggan and Jackson (2006). In Besley and Coate's (1997) citizen candidate model we are faced with the problem that equilibrium sets are potentially very large. The key to equilibrium existence in Roemer (1999) is the assumption that parties have incomplete preferences. In Roemer's (1999) model any party consists of three factions called the opportunists, militants and reformists. Parties only deviate from their platform if all three factions unanimously agree that such a deviation implies an improvement for the party. In the context of the present model an equilibrium would be trivially obtained when each party announces the ideal point of its militant faction, as this faction would veto any deviation from this platform. Roemer avoids such trivial conclusions by imposing some additional assumptions on party preferences that are very reasonable in the context of progressive taxation, the subject of his study. In the very general context of the present study it would be difficult to identify such additional assumptions. It should be said, however, that our assumption on party preferences could be derived from a similar setup as Roemer's (1999) if one where to attribute the different priors in the utility function of a party to different factions within the party. Finally Duggan and Jackson (2006) show that Simon and Zame's (1990) result on endogenous sharing rules can be applied to the case of platform positioning in multidimensional issue spaces. Duggan and Jackson show that mixed strategy equilibria exist in the Downsian model with multiple issues if voters are not assumed to exogenously randomize between two candidates when they are indifferent. What is more Duggan and Jackson show that the supports of the equilibrium strategies must lie in the deep uncovered set.

2 Political Competition

We model political competition as a two stage game played by two different types of actors, two political parties and a large set of voters. First the two parties simultaneously choose their platforms within some (non-empty) convex issue space $X \subset \mathbb{R}^d$, $d \geq 1$. Then the voters, whose preferences are defined over that same issue space X, cast their votes. We assume throughout that parties credibly commit to their platforms and that voters only care about platforms. In particular, no voter has any ideological attachment or bias towards either party.

2.1 The Voters

Throughout this paper we assume that each voter's preferences can be represented by a utility function $u_a^g : X \to \mathbb{R}$ with

$$u_a^g(x) := -\sum_{i=1}^{d} g_i(|x_i - a_i|) \text{ for all } x \in X$$

where $a \in X$, $g_i : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly increasing continuous function for all $1 \leq i \leq d$, and $g := (g_1, ..., g_d)$. We normalize $g_i(0) = 0$ for all i. Since $u_a^g(x)$ is maximized at $x = a$, we call the vector $a \in X$ voter's **ideal point**. Observe that the functions $u_a^g(., x_{-i})$ are single peaked for any $x_{-i} \in X_{-i}$. In fact any preference relation that can be represented by a function u_a^g is single peaked according to the most common notions of single peakedness for multidimensional issue spaces (cf. Barbera, Gul and Stachetti (1993) and Roemer (2001)). The vector g determines the shape of a voter's indifference curves. We refer to the vector g as the **shape** of the voter's indifference curves, we call the function g_i the **attitude** of a voter towards issue i. Geometrically the indifference curves of two voters with the same shape are translations of each other.

Observe that the standard model, in which a voter's utility of a platform is a negative transformation of the Euclidean distance between his ideal point and that platform, is a special case of the present model. To obtain this special case we need to set $g_i(t) := t^2$ for all $t \in \mathbb{R}_+$ and $1 \leq i \leq d$; we denote this shape by g°. The shape g for which $g_i(t) := \alpha_i t$ for some $\alpha_i > 0$ for all $1 \leq i \leq d$ will play a major role in this paper. We denote such a shape by the vector $\alpha := (\alpha_1, ..., \alpha_d)$. The indifference curves of a voter u_a^α are diamond-shaped. Notably, for such a voter the marginal rate of substitution between all issues is constant.

Insert Figure 1

Formally speaking, a **voter** in the present setup is characterized by a pair

$$(a, g) \in X \times H,$$

where X is the issue space and H is some finite set of shapes $H := \{g^1, ..., g^m\}$ with $g^j = (g_1^j, ..., g_d^j)$, each g_i^j being some strictly increasing continuous map on $\mathbb{R}_+$. Consequently, the entire electorate can be described by a distribution of voters (a, g), which is modeled as a Borel probability measure on $X \times H$. Thus, if we denote the set of all Borel probability measures on a metric space Y by $\mathbb{P}(Y)$, then the space of all possible electorates in this model is $\mathbb{P}(X \times H)$.[1] For any $\psi \in \mathbb{P}(X \times H)$, we define the **distribution of voter ideal points** $\psi_a \in \mathbb{P}(X)$ and the **distribution of shapes** $\psi_g \in \mathbb{P}(H)$ as the marginal distributions of a and g, respectively. That is,

$$\psi_a(A) := \psi(A \times H) \quad \text{and} \quad \psi_g(K) := \psi(X \times K),$$

for any Borel sets A in X and K in H.

The expression $\pi_\psi(x, y)$ denotes the share of voters that will vote for platform x given that the other platform is y and given that the electorate is described by ψ. This share $\pi_\psi(x, y)$ can be calculated as the total mass of all voters who strictly prefer x to y plus one

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[1] We use the product metric to metrize $\mathbb{R}^d \times H$ (with H being endowed with the discrete metric). In this case the Borel $\sigma-$algebra on $\mathbb{R}^d \times H$ is equal to the product $\sigma-$algebra on $\mathbb{R}^d \times H$, where $\mathbb{R}^d$ is endowed with the Borel $\sigma-$algebra and H by the algebra of all subsets.

half the mass of all indifferent voters.[2] The following example provides a concrete illustration of the model presented so far.

Example 1: Consider a world in which there are 5 voters and 2 issues. The issue space X equals $[0,1]^2$. There are two voters with ideal point $(\frac{1}{2},\frac{1}{2})$ and shape $\alpha := (1,1)$. There is one voter with ideal point $(0,0)$ and circular indifference curves, two other voters with the same ideal point $(\frac{2}{3},\frac{1}{3})$ but different shapes, one has shape α and the other has circular indifference curves. This electorate is described by the distribution $\psi \in \mathbb{P}([0,1]^2 \times \{g^\circ, \alpha\})$ with $\psi(\{((\frac{1}{2},\frac{1}{2}),\alpha)\}) = \frac{2}{5}$, $\psi(\{((0,0),g^\circ)\}) = \psi(\{((\frac{2}{3},\frac{1}{3}),\alpha)\}) = \psi(\{((\frac{2}{3},\frac{1}{3}),g^\circ)\}) = \frac{1}{5}$. Now suppose the two parties propose the platforms $x = (0,\frac{1}{3})$ and $y = (\frac{5}{6},0)$ respectively. In this case we have that $\pi_\psi(x,y) = \psi((\frac{1}{2},\frac{1}{2}),\alpha) + \psi((0,0),g^\circ) = \frac{3}{5}$.

Insert Figure 2

The dotted lines in Figure 2 represent the indifference curves of the voters $((\frac{2}{3},\frac{1}{3}),\alpha)$ and $((\frac{2}{3},\frac{1}{3}),g^\circ)$ through their preferred platform $(\frac{5}{6},0)$. The dashed arc represents the indifference curve of a voter $((0,0),g^\circ)$ through his preferred platform $(0,\frac{1}{3})$. The remaining (bold) lines all belong to the indifference curve of voter $((\frac{1}{2},\frac{1}{2}),\alpha)$ through his preferred platform: $(0,\frac{1}{3})$.

2.2 The Parties

We maintain the Downsian assumption that the goal of each party is to maximize its vote share. A party's strategy variable is its platform, and hence the issue space X is its strategy space. If the electorate were known, the objective of a party would simply be to maximize $\pi_\psi(x,y)$. The parties in our model, however, do not know the electorate; they are instead uncertain about the preferences of the voters. The innovation of this paper is to study such an environment under the hypothesis that parties are uncertainty averse.

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[2] More formally we can express the vote share-function $\pi_\psi : X \times X \to [0,1]$ by

$$\pi_\psi(x,y) := \psi(T(x,y)) + \frac{1}{2}\psi(I(x,y)).$$

with

$$T(x,y) := \{(a,g) \in X \times H : u_a^g(x) > u_a^g(y)\}$$

and

$$I(x,y) := \{(a,g) \in X \times H : u_a^g(x) = u_a^g(y)\}.$$

for any $(x,y) \in X \times X$, the sets of voters that strictly prefer x to y and the set of indifferent voters, respectively. The vote share function π_ψ is welldefined, since both T and I are measurable. To see this, define $f^j : X \to \mathbb{R}$ by $f^j(a) := u_a^{g^j}(x) - u_a^{g^j}(y)$. Then $T(x,y) = \bigcup_{j=1}^m (\{a \in X : f^j(a) > 0\} \times \{g^j\})$. As all g^j are continuous, all f^j are continuous. Consequently, all sets $\{a \in X : f^j(a) > 0\} \times \{g^j\}$ and hence the set $T(x,y)$ are Borel-measurable. The measurability of $I(x,y)$ is established similarly.

We assume in particular that party preferences can be represented with Maxmin expected utilities following Gilboa and Schmeidler (1989). The preferences of party 1 can be represented by a map $\Pi_{\mathcal{P}} : X \times X \to \mathbb{R}$ with

$$\Pi_{\mathcal{P}}(x, y) := \min_{p \in \mathcal{P}} \int\limits_{\mathbb{P}(X \times H)} \pi_{\psi}(x, y) p(d\psi),$$

where $\mathcal{P} \subseteq \mathbb{P}(\mathbb{P}(X \times H))$ is a non-empty, convex and compact set (of priors on the electorate ψ) and where x denotes the platform of party 1 and y the other party's platform (exchanging the names we obtain party 2's preferences). Furthermore, we assume that parties believe that the voter ideal point distribution is non-atomic and has full support. Formally, we assume that for any $\psi \in$supp(p) for some $p \in \mathcal{P}$, the distribution of voter ideal points ψ_a is non-atomic and supp$(\psi_a) = X$, we write the set of all Borel measures fulfilling these additional assumptions as $\mathbb{P}^*(X \times H)$. In the context of large electorates this seems to be a reasonable simplification.

Note that the set of all electorates $\mathbb{P}^*(X \times H)$ represents the set of all states in this context. Keeping the platform of party 2 fixed at some $y \in X$, party 1 can choose any platform x in X. Each of these platforms x corresponds to an act f_x assigning to every state ψ a consequence $\pi_{\psi}(x, y)$. We impose here that utility party 1 derives from consequence $\pi_{\psi}(x, y)$ is again $\pi_{\psi}(x, y)$, so the function $\psi \mapsto \pi_{\psi}(x, y)$ represents the utility of party 1 in each state. In other words we assume that parties are vote share maximizers.[3] A generic element of the set $\mathbb{P}(\mathbb{P}^*(X \times H))$ is viewed as a belief (or prior) on the electorate; it is a probability measure on the set of all electorates (or states) $\mathbb{P}^*(X \times H)$.

Schmeidler's (1989) concept of uncertainty aversion, which requires that an uncertainty averse agent who is indifferent between two acts (weakly) prefers a mixture of these two acts to either one of them, can be expressed as follows in the context of the game of platform positioning: Suppose party 1 is indifferent between offering a platform x and a platform x' in the election. Now let that party go through the following thought experiment: the party considers to face that same identical election twice. In each of these identical elections party 2 offers the fixed platform y. Then, by the uncertainty aversion axiom of Gilboa and Schmeidler, party 1 must like to offer x in one of the elections and x' in the other at least as much as offering x in both elections. In short, we assume that if a party had a chance to hedge their bets, they would not reject this chance.

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[3] This assumption on the preferences of parties is standard but not without criticism. The other most common assumption on party prefernces is that they maximize their probability of winning. The true goal of a party is likely a convex combination of these two. We chose the assumption of vote share maximization since it allows for some neat simplifications of the parties' objective functions.

The utility representation of a vote share maximizing party can be simplified quite a bit. It turns out that it is equivalent to think of the decision-problem of parties in terms of expected vote shares or in terms of vote shares according to the expected electorate. More formally, we have

Proposition 1: *For any non-empty, convex and compact subset* $\mathcal{P}$ of $\mathbb{P}(\mathbb{P}^*(X \times H))$, *the set* $\Psi := \{\int_{\mathbb{P}^*(X\times H)} \psi p(d\psi) : p \in \mathcal{P}\}$ *is a convex subset of* $\mathbb{P}^*(X \times H)$, *and*

$$\Pi_{\mathcal{P}}(x, y) = \min_{\psi\in\Psi} \pi_\psi(x, y) \quad \textit{for all } (x, y) \in X^2.$$

Proof: See Appendix. ■

Proposition 1 shows that it is without loss of generality to represent the preferences of party 1 over platform profiles by a map of the form:

$$(x, y) \mapsto \min_{\psi\in\Psi} \pi_\psi(x, y)$$

where Ψ is some non-empty convex subset of $\mathbb{P}^*(X \times H)$. Moreover, it will simplify things greatly to view Ψ as the primitive of the model and to introduce our further assumptions on party beliefs as postulates on this set. (The elements of Ψ are henceforth referred to as *beliefs about electorates, electorates* and *expected electorates* interchangeably.)

*Remark:***.** Expected vote share maximizing parties arise as a special case of multiple-prior formulation of party preferences. If the set of priors $\mathcal{P}$ is a singleton, say $\{p^*\}$, the objective of any party is to maximize its expected vote share. In this case the utility of party 1 reduces to

$$\Pi_{\mathcal{P}}(x, y) = \int\limits_{\mathbb{P}^*(X\times H)} \pi_\psi(x, y) p^*(d\psi).$$

Following Proposition 1 this is none other than the vote share of party 1 when the platform profile is (x, y) given the expected electorate $\psi^* := \int_{\mathbb{P}^*(X\times H)} \psi p(d\psi)$. That is,

$$\Pi_{\{p^*\}}(x, y) = \pi_{\psi^*}(x, y)$$

where $\psi^*(S) := \int_{\mathbb{P}^*(X\times H)} \psi(S)p(d\psi)$ for all Borel subsets S of $X \times H$. Therefore, models with parties that are certain about the electorate and models with expected vote share maximizing parties are formally equivalent. Consequently, there is no hope to solve the well-known nonexistence problems of the standard electoral competition models with multidimensional issue spaces by introducing risk (or Savagean uncertainty) into these models.

2.3 The Voting Game

A two player game of political competition is then characterized by the triplet (d, X, Ψ), where d is the dimension of the issue space X and $\Psi = (\Psi_1, \Psi_2)$ is the set of beliefs of parties 1 and 2. This is a normal-form game in which the action space of either of the parties is X, and where the payoff functions of party 1 and 2 are defined as the following maps on X^2:

$$(x, y) \mapsto \min_{\psi \in \Psi_1} \pi_\psi(x, y) \quad \text{and} \quad (x, y) \mapsto \min_{\psi \in \Psi_2} (1 - \pi_\psi(x, y)) = \min_{\psi \in \Psi_2} \pi_\psi(y, x).$$

To save on notation we let $\Psi_1 = \Psi_2 = \Psi$. This assumption is not needed for any of the results in the paper: all results hold if the conditions on Ψ are instead applied to each of the set of beliefs Ψ_1, Ψ_2. The set Ψ does not only reflect the parties' beliefs about the electorate but it also reflects their degree of uncertainty aversion. Therefore it is not appropriate to invoke the common prior assumption as a justification of the assumption that $\Psi_1 = \Psi_2 = \Psi$. Two actors that have access to the same amount of information might base their decisions on different sets of priors, simply because one might be more uncertainty averse than the other. There are some decision theoretic models that distinguish between the uncertainty of the decision maker and his attitude towards that uncertainty; for instance. Ghiradato (2004) and Klibanoff, Marinacci and Mukerji (2005). For soe further discussion on the common prior assumption see remark d) in Section 4.1.

We need further assumptions on the nature of the set Ψ of beliefs of the parties to establish our main results. To this end, we define the set $\Psi(\Lambda, G)$ as the set of beliefs on the electorate that has been generated by the set Λ of voter ideal point distributions and by the set G of shapes as follows:

$$\Psi(\Lambda, G) := \{\psi \in \mathbb{P}^*(X \times H) : \psi_a \in \Lambda \text{ and } \operatorname{supp}(\psi_g) \subseteq G\}.$$

This definition says that for any electorate in $\Psi(\Lambda, G)$, the voter ideal point distribution belongs to Λ and the shape of the indifference curves of any voter belongs to the set G. Our assumption states that party beliefs about the electorate can be generated from a set of beliefs on the voter ideal point distribution and some sets of attitudes towards all the different issues.

(A1) There exists a non-empty, convex set $\Lambda \subset \mathbb{P}(X)$ and non-empty, finite sets G_i of strictly increasing and continuous maps on $\mathbb{R}_+$ such that $\Psi = \Psi(\Lambda, G_1 \times \cdots \times G_d)$.

This assumption can be interpreted as an independence assumption. To see this imagine that X would be finite. Assume that parties would independently form beliefs on the distribution of voter ideal points μ on X and on the conditional distribution of attitudes towards single issues $\psi_{g_i|x}$. If this were the case, the set of expected electorates would be representable as the product of a set of (expected) distributions on X with a set of expected conditional probabilities $E(\psi_{g_i|x})$. Any belief set on the electorate that can be constructed from such *basic* beliefs about the voter ideal point distributions and the conditional distribution of shapes would satisfy (A1). The reason why this assumption is not derived from a basic assumption on independent belief formation is the presence of complicated issues of measurability when considering an infinite set X. These arguments are spelled out further in the appendix.

(A1) excludes the case that certain attitudes about issues say g_1' only arise in combination with certain other attitudes about other issues, say g_2' or g_2''. To the contrary if according to some belief the indifference curves of some voters have the shape (g_1, g_2) (over a two dimensional issue space) while others have the shape (g_1', g_2'), then there must be an alternative belief in the set Ψ according to which some voters have indifference curves with shape (g_1', g_2). (A1) also excludes the case that parties associate certain sets of ideal points with a particular type of indifference-curve shape. (A1) requires that if for some $\psi \in \Psi$ the parties believe that all voters with ideal points in a some set $A \subset X$ have the shape g, then there must be a $\psi' \in \Psi$ such that parties believe that all voters with ideal point not in A have that shape g.

The standard case of an electorate where all voters are assumed to have Euclidean preferences fulfils (A1), as can be seen by letting G_i consist only of the map $t \mapsto t^2$ for all i. The belief set of parties that are maximally uncertain about the electorate also satisfies (A1) as we have that $\mathbb{P}^*(X \times H) = \Psi(\mathbb{P}^*(X), H)$. More importantly, given (A1), we can control the amount of uncertainty in the model, by varying the sizes of Λ and G. For instance, take a set of belief sets $\Psi^\nu := \Psi(\Lambda^\nu, G^\nu)$ for a sequence of sets $(\Lambda^\nu, G^\nu)_{\nu \in \mathbb{N}}$. Let $\Lambda^\nu \supset \Lambda^{\nu+1}$ and $G^\nu \supset G^{\nu+1}$ for all $\nu \in \mathbb{N}$. Then the uncertainty of parties decreases as ν becomes larger. Consequently the model's assumptions on uncertainty and the aversion against it are not extreme, to the contrary the degree of uncertainty can be varied continuously within the model.

For any voter ideal point distribution $\mu \in \mathbb{P}(X)$ and any shape α, there exists a (unique) electorate ψ for which we have that $\psi_a = \mu$ and $\psi_g(\alpha) = 1$. In words, there is a unique electorate in which all voters have indifference curves of the same (diamond-)shape α and for which the voter ideal point distribution is μ. We denote this electorate by $\mu \times \delta_\alpha$, that is $\{\mu \times \delta_\alpha\} = \Psi(\{\mu\}, \{\alpha\})$.[4] Electorates of this kind will play a major role in our proofs.

Under the Assumption (A1) we can represent the present political competition game as the quadruple (d, X, Λ, G), where Λ denotes the set of all of the parties' priors on the

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[4] The expression $\delta_\alpha \in \mathbb{P}(H)$ denotes the Dirac measure that assigns probability 1 to the trade-off pattern α.

distribution of voter ideal points and $G := G_1 \times \cdots \times G_d$ denotes the finite set of shapes that parties consider. We refer to this game as a **voting game with uncertainty averse parties,** and note that, formally speaking, we have

$$(d, X, \Lambda, G) := (d, X, \Psi(\Lambda, G)).$$

We denote the set of political equilibria in this game by $\mathcal{E}(d, X, \Lambda, G)$, where a (**political) equilibrium** is a Nash equilibrium of $(d, X, \Psi(\Lambda, G))$, that is, $(x, y) \in \mathcal{E}(d, X, \Lambda, G)$ if and only if

$$x \in \underset{z \in X}{\arg\max} \left(\min_{\psi \in \Psi(\Lambda, G)} \pi_\psi(z, y) \right) \quad \text{and} \quad y \in \underset{z \in X}{\arg\max} \left(\min_{\psi \in \Psi(\Lambda, G)} (1 - \pi_\psi(x, z)) \right).$$

Remarks:. (a) We are only looking at pure strategy equilibria. Pure strategy equilibria have the appeal that neither party has an incentive to deviate once they have observed the platform of the other. Pure strategy equilibra are therefore robust to different assumptions on the timing of the game of platform positioning.

(b) Observe that parties are assumed to be uncertain about the environment in the present model. They are not assumed to be uncertain about each others strategies. There are three reasons for this assumption. First of all, since we are looking only at pure strategies, it is not clear how "strategic uncertainty" should be interpreted in the present context. Secondly we could imagine a gradual process in which parties move their platforms closer and closer, with the equilibrium as a resting point. In this process either party would learn about the other's beliefs on the electorate through the strategic choices of the other party. They would not learn as much about the electorate, since the electorate only votes once the process of platfrom positioning is concluded. Thirdly, while the equilibrium notion used in the present context arises out of a straightforward application of Nash equilibrium, there is no canonical way to define equilibrium for the context of uncertainty about the other players strategies. For a review of the opposing views on the appropriate equilibrium concept in such a context see Mukerji and Tallon (2004).

3 Uncertainty about Shapes of Indifference Curves

In this section we study games in which parties are uncertain about the shapes of the voter's indifference curves, while being fully informed about the distribution of ideal points (or, equivalently, act as expected utility maximizers with respect to the distribution of voter ideal points). Such a game is modeled as $(d, X, \{\mu\}, G)$. We first restrict our attention to this *one* type of uncertainty to see what degree of shape-uncertainty alone suffices to obtain political equilibria. Note that with the assumption that there is no subjective uncertainty concerning the distribution of the electorate we are able to use the common prior assumption and impose that both parties share the same belief μ on the distribution of voter ideal points.

The marginal rate of substitution between two issues can - for any given platform and ideal point - be determined from the shape of a voter's indifference curves. Consequently we can interpret uncertainty about the shape as uncertainty about the marginal rates of substitution. The condition we are studying here is that for any platform the parties are uncertain as to whether the marginal rate of substitution of a voter is more or less than some fixed rate. In other words, we will assume that for any issue $i \neq 1$ there exists a rate of substitution β_i such that parties are uncertain whether a voter's marginal rate of substitution between that issue i and issue 1 is smaller or larger than β_i. Accordingly for any two issues i and j parties are uncertain if the marginal rate of substitution between those two issues is smaller or larger than $\frac{\beta_i}{\beta_j}$.

A convenient but different way to express this assumption is:

Definition: A party is **(strictly) uncertain about the marginal rates of substitution** if there exists an $\alpha \in \mathbb{R}^d_{++}$ such that in all G_i there are some differentiable functions h_i and k_i with $h_i'(t) \geq \alpha_i \geq k_i'(t)$ for all $t \in \mathbb{R}_+$ (or in the case of strict uncertainty $h_i'(t) > \alpha_i > k_i'(t)$).

To illustrate this condition consider a voter with ideal point a and some platform x. While parties in the present model do not know that voter's indifference curve through platform x, we do assume that they know that this indifference curves lies within certain bounds. Also observe that, if parties are uncertain about the salience of all issues, we can always renormalize the sets G_i such that $\alpha_1 = 1$.

Alternatively we could assume that voters have thick indifference curves and each party assumes that all indifferent voters and all voters with a strict preference for the other party will vote for the other party. Now, if there exists a shape α such that every of these thick indifference curves contains the indifference curve of that shape α, the parties are uncertain about the marginal rates of substitution according to our definition. This definition is not vacuous. If all voters have Euclidean preferences, the condition is for example not fulfilled. Observe on the other hand, that parties need not be very uncertain for this condition to be fulfilled. Indifference curves do not have to be very thick to contain the indifference curves of shape α. Parties might for example know that the marginal rate of substitution between issues i and j lies in a small interval around $\frac{\alpha_i}{\alpha_j}$. In fact, parties need not be uncertain at all to fulfill our definition of uncertainty about the marginal rates of substitution. To see this, consider the extreme case that parties know that all voters' indifference curves have the same shape α. For such voters the marginal rates of substitution between all issues are constant over the entire range of platforms.Our definition of uncertainty about marginal rates of substitution still applies, even though the set G and therefore also the set $\Psi(\{\mu\}, G)$ are singletons.

Theorem 1. *Let* $(2, X, \{\mu\}, G)$ *be a game where parties are uncertain about the marginal rates of substitution. Then* $(2, X, \{\mu\}, G)$ *has a political equilibrium.*

There is no hope of demonstrating this theorem using Nash's Existence Theorem or any of its relatives, for the best response correspondences in games of the type $(2, X, \{\mu\}, G)$ are generally not convex-valued. We proceed by means of a different strategy: we first show that there is only one candidate for an equilibrium, and then prove that in our game there does not exist any preferred deviation for either party from that platform profile.

3.1 Characterization of Equilibria

To characterize the set of equilibria of a voting game $(d, X, \{\mu\}, G)$, we need to introduce the notion of the median vector. For any probability-distribution μ, we call the vector of the medians of all marginal distributions μ_i, that is the vector $(m(\mu_1), ..., m(\mu_d))$, the **median vector** of μ. We denote this by $\mathbf{m}(\mu)$. Throughout this section we normalize the median vector of μ to $\mathbf{0} : \mathbf{m}(\mu) = \mathbf{0}$,and take the convention that $\mathbf{0}$ denotes the vector $(0, 0, ..., 0)$ (the dimension of this vector will always be clear from the context). The uniqueness of the median vector follows from the assumption of μ having full support on the convex set X.

Proposition 2. *Let* (x, y) *be an equilibrium of a game* $(d, X, \{\mu\}, G)$, *then* $x = y = \mathbf{0}$.

Proof: Suppose that $(x, y) \neq (\mathbf{0}, \mathbf{0})$, say $y_1 \neq 0$. Let $x' := (0, y_2, ..., y_d)$. Facing the choice between x' and y, all voters decide on the basis of whether $g_1(|0 - a_1|)$ is smaller or larger than $g_1(|y_1 - a_1|)$. Since 0 is median of μ_1, at least half of the voters will vote for x'. And since supp(μ) is convex and since G_1 is a finite set of continuous functions, we have $\pi_\psi(x', y) > \frac{1}{2}$ for all $\psi \in \Psi(\{\mu\}, G)$. Consequently, $\min\limits_{\psi \in \Psi(\{\mu\}, G)} \pi_\psi(x', y) > \frac{1}{2}$. Since x is a best response to y, in equilibrium party 1 receives a utility of $\min\limits_{\psi \in \Psi(\{\mu\}, G)} \pi_\psi(x, y) > \frac{1}{2}$. Therefore party 2 only receives a utility of $\min\limits_{\psi \in \Psi(\{\mu\}, G)} (1 - \pi_\psi(x, y)) < \frac{1}{2}$ in equilibrium. But then party 2 would be strictly better off by changing its platform to x since $\min\limits_{\psi \in \Psi(\{\mu\}, G)} (1 - \pi_\psi(x, x)) = \frac{1}{2}$. ■

The intuition behind Proposition 2 is that any party can choose to compete with the other in only one dimension. The only protection against such "one-dimensional attacks" is to propose the pertaining median $m(\mu_i) = 0$ with respect to every issue i. Also observe that the certainty case is covered by Proposition 2: G might be a singleton. If a distribution μ has a generalized median it coincides with the median vector. We now turn to the problem of the existence of equilibrium. Note that we did not use the fact that we equalized both parties belief sets on the shapes G.

3.2 The Existence of Equilibrium

A sufficient condition for Theorem 1 to hold is that for any deviation y from the median vector $\mathbf{0}$, there exists an electorate ψ^y in the set of electorates $\Psi(\{\mu\}, G)$, such that $\pi_{\psi^y}(y, \mathbf{0}) < \frac{1}{2}$. If there is one such electorate, then $\min\limits_{\psi \in \Psi(\{\mu\},G)} \pi_\psi(y, \mathbf{0}) < \frac{1}{2}$ while $\min\limits_{\psi \in \Psi(\{\mu\},G)} \pi_\psi(\mathbf{0}, \mathbf{0}) = \frac{1}{2}$. Uncertainty aversion proves a strong force towards both parties announcing the same platform. The entire uncertainty is eliminated when both parties announce the same platform since in this case they both receive half the vote under *any* assumption on the electorate.

The complete proof that such a ψ^y exists can be found in the appendix; here we give a sketch of the basic argument. First we show that, given our assumption that parties are uncertain about the marginal rates of substitution, for any platform $y \in X$ there exists some $\psi^y \in \Psi(\{\mu\}, G)$ such that $\pi_{\psi^y}(y, \mathbf{0}) \leq \pi_{\mu \times \delta_\alpha}(y, \mathbf{0})$ where α is chosen such that there exist $h_i, k_i \in G_i$ with $h_i'(t) \geq \alpha_i \geq k_i'(t)$ for all $t \in \mathbb{R}_+$ and $i = 1, 2$. So while the shape α itself might not be contained in G, for any y there exists some electorate $\psi \in \Psi(\{\mu\}, G)$ according to which the vote share of party 1 is no higher than what it would get in the case that all voters had shape α. In fact, this holds independently of the dimension of the issue space of the game, and we conclude that whenever $(\mathbf{0}, \mathbf{0})$ is an equilibrium of the a game $(d, X, \{\mu\}, \{\alpha\})$, it is also an equilibrium of the game $(d, X, \{\mu\}, G)$ for any $d \in \mathbb{N}$.

The last step of the proof builds on a fact that was discovered by Rae and Taylor (1971): Their observation that any game $(2, X, \{\mu\}, \{\alpha\})$ has an equilibrium completes our proof. It shoudl be noted that the present result is slightly different from Rae and Taylor's (1971) resul as they assumed a finite and odd number of voters. We provide our own proof of this very similar result for two reasons: first we consider infinite electorates, secondly and more importantly the argument in Rae and Taylor (1971) is flawed insofar as they claim that "the proof of this theorem [the existence of an equilibrium in the two-dimensional case] for any number of dimensions is immediate." Below (example 2) we provide a counterexample to this claim.

Without loss of generality, we only investigate deviations $y \gg \mathbf{0}$. First we establish that for any deviation with $y_1\alpha_1 \neq y_2\alpha_2$, less than half of the electorate votes for the deviator; in this case the partisans of the deviation can either be all found above the median line $x_2 = 0$ or to the right of the median line $x_1 = 0$. (The dashed lines in Figure 3 and 4 show the set of all indifferent voters for different deviations y, and different shapes α. In each case the dashed lines separte the set of voters with a strict preference for y from the set of voters with a strict preference for $\mathbf{0}$. Note that in either case this set is a strict subset of a "half" of the electorate, as described by the horizontal and vertical lines through **0.)**

Insert Figures 3 and 4

Finally, for the case of $y_1\alpha_1 = y_2\alpha_2$ the set of voters who prefer y to $\mathbf{0}$ is a subset of the positive quadrant while the set of voters preferring $\mathbf{0}$ to y is a superset of the negative quadrant. The observation that the positive and the negative quadrant of any two dimensional distribution (with median vector $\mathbf{0}$) must contain an equal amount of probability mass concludes the proof. (The dotted areas in Figure 5 represent the set of indifferent voters.It is important to note that the set of voters strictly preferring the deviation y is s subset of the upper right quadrant, whereas the set of voters strictly preferring $\mathbf{0}$ is a superset of the lower left quadrant.)

Insert Figure 5

So $(\mathbf{0}, \mathbf{0})$ is indeed an equilibrium of the game $(2, X, \{\mu\}, G)$ when parties are uncertain about the marginal rates of substitution.

Remarks: Observe that we do not use the "common prior" assumption in this proof. The proof would also hold true if both parties where to hold different beliefs G as long as each of the parties fulfills the assumption of uncertainty about the marginal rates of substitution (two different α's can be used). The only reason for the assumption of a "common prior" is that it allows us to reduce on notation. The same holds true for the results in the reminder of the paper.

Unfortunately, the result that any game $(2, X, \{\mu\}, \{\alpha\})$ has an equilibrium, does not extend to higher dimensions. If it did, we could apply the proof of Theorem 1 to games of any dimension, since the two-dimensionality of the game was used only to establish that any game $(2, X, \{\mu\}, \{\alpha\})$ has an equilibrium. To obtain existence results for higher dimensional issue spaces, it appears that we need to introduce some uncertainty about the distribution of ideal points. But before doing so, let us conclude this section by providing an example of a three-dimensional game that does not have an equilibrium even though the parties are uncertain about the marginal rates of substitution. Observe, that this example at the same time shows that the games $(d, X, \{\mu\}, \{\alpha\})$ might have no equilibria for $d > 2$.

Example 2. Consider the game $(3, X, \{\mu\}, \{\alpha\})$, with $X = [-1, 1]^3$ and μ given by the following chart, where the left column denotes subspaces of $[-1, 1]^3$ and the right column the probability mass in those subspaces. We assume that the conditional distribution in any of the subspaces S is uniform.

S	$\mu(S)$
$[0,1]^3$	.3
$[0,1]^2 \times [-1,0)$	.05
$[0,1] \times [-1,0) \times [0,1]$	.05
$[0,1] \times [-1,0)^2$	.1
$[-1,0)^3$	.2
$[-1,0)^2 \times [0,1]$	.15
$[-1,0) \times [0,1] \times [-1,0)$	.15
elsewhere	0

Assume that $\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{3}$, that is, assume that voters care equally much about all three issues. Observe that the median vector is at the origin, $\mathbf{m}(\mu) = \mathbf{0}$, and therefore, by Proposition 2, $(\mathbf{0}, \mathbf{0})$ is the only candidate for any equilibrium. But $(-\frac{1}{1000}, -\frac{1}{1000}, -\frac{1}{1000})$ is a preferred deviation from this platform profile as nearly all voters with ideal points in $[-1,0)^3$, $[-1,0)^2 \times [0,1]$, $[-1,0) \times [0,1] \times [-1,0)$ and $[0,1] \times [-1,0)^2$ will vote for the deviator, and these sets make up 60% of the electorate. All voters in these quadrants that are sufficiently far away from the origin prefer $(-\frac{1}{1000}, -\frac{1}{1000}, -\frac{1}{1000})$, as this platform is closer to their ideal point with respect to at least two dimensions. Since there are only very few voters in the immediate proximity of $(-\frac{1}{1000}, -\frac{1}{1000}, -\frac{1}{1000})$ and $\mathbf{0}$ this argument concerns nearly 60% of the electorate.

4 Uncertainty about the distribution of ideal points

In this section, we consider the case that parties are uncertain about the distribution of ideal points μ. This will not only add to more realism of the model, it will also allow us to extend our prior 2-dimensional existence result (Theorem 1) to higher dimensions. In particular, the main result of this section is that if the two uncertainty averse parties do not know whether the electorate leans left or right (in a sense to be defined precisely below), and are also uncertain about the marginal rates of substitution, then political equilibria exist in voting games with 3-dimensional issue spaces.

4.1 Characterization of equilibria

Median vectors will again play an important role in the characterization of equilibria. In the present case, however, there is no *one* median vector, for parties hold multiple priors on the distribution of voter ideal points. We can characterize the set of possible equilibrium platform profiles as a function of the median vectors of all the electorate that parties take into account. Given a set Λ of voter ideal point distributions on $\mathbb{R}^d$ we, thus, concentrate on the **median set** $M(\Lambda)$, which is defined as

$$M(\Lambda) := \left\{ x \in \mathbb{R}^d : \min_{\mu\in\Lambda} m(\mu_k) \leq x_k \leq \max_{\mu\in\Lambda} m(\mu_k) \text{ for all } 1 \leq k \leq d \right\}.$$

This can be thought of as the set of all platforms *in-between* the median platforms of the different distributions μ in Λ. The counterpart of Proposition 2 in the present context reads as follows:

Proposition 3: *Let (x, y) be an equilibrium of a game (d, X, Λ, G), then $x, y \in M(\Lambda)$.*

Proof: Suppose that $y \notin M(\Lambda)$, say $y_1 < \min_{\mu\in\Lambda} m(\mu_1)$. Then by the same argument as in the proof of Proposition 2, a deviation from x to $x' := (\min_{\mu\in\Lambda} m(\mu_1), y_2, ..., y_d)$ yields for every $\psi \in \Psi(\Lambda, G)$ a vote share $\pi_\psi(x', y) > \frac{1}{2}$. Thus $\min_{\psi\in\Psi(\Lambda,G)} \pi_\psi(x', y) > \frac{1}{2}$ and as x is a best response to y, the utility of party 1, $\min_{\psi\in\Psi(\Lambda,G)} \pi_\psi(x, y)$, is larger than $\frac{1}{2}$. So, in equilibrium, party 2 only obtains a utility of $\min_{\psi\in\Psi(\Lambda,G)} (1 - \pi_\psi(x, y)) < \frac{1}{2}$, and hence it would be strictly better off to change its platform to x as $\min_{\psi\in\Psi(\Lambda,G)} (1 - \pi_\psi(x, x)) = \frac{1}{2}$. ■

Remarks: (a) In passing, we note that we do not know if in equilibrium both parties need to announce the same platform. In Bade (2004) I show that if we model uncertainty aversion in a different way following Bewley (2002) equilibria with both parties announcing different platforms should always be expected to arise.

(b) It may be tempting to presume that in any equilibrium both parties must announce the median vector of some distribution in Λ. It is easy to show that this is not true. Consider the game $(2, X, \Lambda, \{\alpha\})$ where $\Lambda := \{\lambda\nu + (1-\lambda)\phi : 0 \leq \lambda \leq 1\}$, and assume that $\text{supp}(\nu) = \text{supp}(\phi) = [-1, 2]^2$, $\nu([-1, 1]^2) = \phi([0, 2]^2) = .999$, and that the distribution of ν conditional on $[-1, 1]^2$ as well as the distribution of ϕ conditional on $[0, 2]^2$ are uniform. Let us finally also impose that $m(\nu_1) = m(\nu_2)$ and $m(\phi_1) = m(\phi_2)$. Then $((\frac{1}{3}, \frac{2}{3}), (\frac{1}{3}, \frac{2}{3}))$ is an equilibrium of this game, even though $m(\mu_1) = m(\mu_2)$ holds for all $\mu \in \Lambda$.

(c) The equilibrium platforms depend on the actual preferences of voters only insofar as that beliefs of parties about the electorate are related to the actual preferences of voters. We consider this reasonable: the parties are picking the platforms so the outcome of the game should depend on whatever they know or believe about the electorate. Also observe that there remains empirical tractability to this question. It is to be expected that party beliefs about the locations of all the "median voters" are heavily influenced by or related to publicly available information on the subject such as polls or research studies.

(d) The assumption of common belief sets is not necessary. However, observe that the beliefs ests need to be close enough in the sense that in equilibrium both parties must locate in $M(\Lambda_1)$ as well as in $M(\Lambda_2)$. They can do so if and only if the intersection between these two sets is not empty. If the intersection is empty the beliefs of parties diverge so far as to allow for strategy profiles in which each party is sure it will win the election. Billot et al. (2000) develop a notion of “Sharing Beliefs" that generalizes the idea of common priors to environments with multiple agents that all have maxmin expected utilities following Gilboa and Schmeidler (1989). If the criterion mentioned in Billot et. al. (2000) is satisfied in the present context then the intersection of $M(\Lambda_1)$ and $M(\Lambda_2)$ must be non-empty.

4.2 The Existence of Equilibrium

To state sufficient conditions for the existence of equilibria in games with distribution uncertainty, we have to introduce some more concepts. Let us agree to call a voter a *leftist* if her ideal point lies with respect to *all* issues below the median vector. That is, a voter with ideal point $a \in X$ is a **leftist (rightist)** iff $a_i < m(\mu_i)$ ($a_i > m(\mu_i)$, respectively) for all issues i. Given some distribution μ of voter ideal points, we denote the set of all leftists and rightists by A_l^μ and A_r^μ respectively. A distribution μ is called **left leaning (right leaning)** if $\mu(A_l^\mu) > \mu(A_r^\mu)$ ($\mu(A_r^\mu) > \mu(A_l^\mu)$ respectively). Finally, a distribution μ with equally many leftists and rightists ($\mu(A_l^\mu) = \mu(A_r^\mu)$) is called **balanced**. Observe that, any 2-dimensional distribution is balanced, in the 2 dimensional case $\mu(A_l^\mu)$ can be calculated as $1 - \mu(a : a\_1 > 0) - \mu(a : a\_2 > 0) + \mu(A_r^\mu) = \mu(A_r^\mu)$. We call a party **uncertain as to whether the electorate leans to the left or to the right** if there exists a left leaning and a right leaning distribution in the set of party beliefs on the distribution of voter ideal points.

Theorem 2: *Let* $(3, X, \Lambda, G)$ *be a game where parties are uncertain about the marginal rates of substitution and uncertain as to whether the electorate leans to the left or to the right. Then* $(3, X, \Lambda, G)$ *has a political equilibrium.*

Theorem 2 generalizes Theorem 1 and its proof is reminiscent of the proof of Theorem 1. We showed already that under the assumption that parties are uncertain about the marginal rates of substitution there exists an $\alpha \in \mathbb{R}^d_{++}$ such that $\mathcal{E}(3, X, \{\mu\}, \{\alpha\}) \subseteq \mathcal{E}(3, X, \{\mu\}, G)$ for any fixed μ. If a deviation from some platform profile is detrimental according to a particular belief $\mu \in \Lambda$, then this deviation is detrimental when taking the entire set of beliefs on the voter ideal point distributions Λ into account. Within the formalism of this paper this argument can be expressed as $\min_{\psi \in \Psi(\Lambda, G)} \pi_\psi(x, y) < \min_{\psi \in \Psi(\{\mu\}, G)} \pi_\psi(x, y)$ for $\mu \in \Lambda$. We therefore have $\mathcal{E}(3, X, \{\mu\}, G) \subseteq \mathcal{E}(3, X, \Lambda, G)$ for all $\mu \in \Lambda$. So if we can show that

$\mathcal{E}(3, X, \{\mu\}, \{\alpha\})$ is non-empty for some $\mu \in \Lambda$, we are done. The main difficulty lies in the fact that $\mathcal{E}(3, X, \{\mu\}, \{\alpha\})$ might be empty (see Example 2). Yet, in the appendix we generalize Rae and Taylor (1971) and show that $(3, X, \{\mu\}, \{\alpha\})$ has an equilibrium if and only if μ is balanced. Given our assumption that the parties are uncertain as to whether the electorate is left or right leaning we can show that there exists some balanced μ^* in Λ, and we thus obtain $\mathcal{E}(3, X, \{\mu^*\}, \{\alpha\}) = (\mathbf{m}(\mu^*), \mathbf{m}(\mu^*))$. It follows that $(\mathbf{m}(\mu^*), \mathbf{m}(\mu^*)) \in \mathcal{E}(3, X, \Lambda, G)$.

The following example shows that the conditions given in Theorem 2 are sufficient but not necessary for the existence of an equilibrium.

Example 3: Let $(3, [-1, 1]^3, \Lambda, G)$ be a game with Λ the set of all convex combinations of the voter ideal point distributions $\mu^1, ..., \mu^8$. Assume that all of these distributions μ^i are right leaning. Assume furthermore that for each of the eight octants around $\mathbf{0}$ there is a distribution μ^i amongst $\mu^1, ..., \mu^8$ such that the interior of that octant contains at least .6 of the probability mass according to μ^i. (Observe that we do **not** require $\mathbf{0}$ to be the median vector of any one of these distributions of voter ideal points.) We claim that $\mathbf{0}$ is an equilibrium of that game. Observe that, by our assumption that all voters' preferences are single peaked, for any deviation y from $\mathbf{0}$ there exists some octant such that all voters in that octant are voting for $\mathbf{0}$. Now, by our construction of the set Λ there exists a μ in it such that at least .6 of all voters prefer $\mathbf{0}$ to y according to that μ. Consequently $(\mathbf{0}, \mathbf{0})$ is an equilibrium even though by construction all distributions $\mu \in \Lambda$ are right leaning. Observe also, that we did not specify G to obtain this result, G might even be a singleton.

Roughly speaking, Theorem 2 establishes that any 3-dimensional game played amongst parties that are neither certain about the salience of issues nor about the leanings of the electorate has an equilibrium. Example 3 shows that this amount of uncertainty is sufficient but not necessary for the existence of equilibria. But, from Proposition 3 we know that in any equilibrium, whether the sufficient conditions are fulfilled or not, both parties have to announce a policy from the median set. This means that with respect to every issue i, parties announce the ideal of a voter that is the median voter of the marginal distribution μ_i of some $\mu \in \Lambda$.

5 Higher Dimensional Issue Spaces

We now turn to discussing the type of conditions that would ensure the existence of equilibria in games of political competition in which the issue space has dimension higher than 3. In fact, given the previous analysis, it is easy to state a condition for the existence political equilibria in $d-$dimensional games with uncertain parties. To this end we need one further bit of notation.

Let us define the function **sgn**: $\mathbb{R}^d \to \{1, 0, -1\}^d$ by $\mathbf{sgn}(x) = (\mathrm{sgn}(x_1), ..., \mathrm{sgn}(x_d))$, and let

$$A_f^\mu := \{x|\mathbf{sgn}(x - m(\mu)) = f\} \text{ for all } f \in \{1, -1\}^d.$$

The expressions A_f^μ describe the "quadrants" around the median vector of a distribution. We call a distribution $\mu \in \mathbb{P}(X)$ **equilibrated**, if there is the same amount of probability mass in each pair of opposing quadrants, that is $\mu(A_f^\mu) = \mu(A_{-f}^\mu)$ for all $f \in \{1, -1\}^d$. Note that any balanced $3-$dimensional distribution and any $2-$dimensional distribution is equilibrated.

Theorem 3: *Let $d \in \mathbb{N}$ and let (d, X, Λ, G) be a game where parties are uncertain about the marginal rates of substitution. If there exists an equilibrated μ in Λ, then (d, X, Λ, G) has an equilibrium.*

In this result we require that there exists some equilibrated μ in Λ whereas Theorem 2 requires that parties do not know whether the electorate leans to the left or to the right. Within 3-dimensional voting games these conditions are closely related: the uncertainty about the leanings of the electorate implies the existence of an equilibrated μ in Λ. With a minor change in our notion of left and right leaning, namely if we call an electorate left (right) leaning if there are *weakly* more leftists (rightists) in the electorate, the two conditions are equivalent for 3-dimensional distributions. Consequently Theorem 3 generalizes Theorem 2 (and thereby Theorem 1) to the context of a d-dimensional issue space.

The proof of Theorem 3 is analogous to that of Theorem 2. In fact, a crucial step in the proof of Theorem 2 was to establish the existence of an equilibrated μ in Λ under the conditions of that theorem. In Theorem 3 we simply impose the existence of such a distribution as a hypothesis. From then on both proofs are identical.

The condition that there exists an equilibrated μ in Λ is, admittedly, not very intuitive. However, it may well be the case that the parties' uncertainty about electorates increases with the dimensionality of the issue space. Perhaps this uncertainty about the distribution of ideal points may indeed be large enough that some equilibrated distributions could be considered a plausible assumption on the voter ideal point distribution by the agenda setters of the parties. At any rate, Theorem 3 (like Theorem 2) provides only sufficient conditions. Example 3 demonstrates that there remains room for weaker conditions that would warrant the existence of equilibrium.

6 Conclusion

In this paper we validate Ordeshook's suggestion that the incorporation of an assumption that "candidates are uncertain are uncertain about voter's exact preferences" into a model of platform positioning can yield stable equilibrium predictions. Given that parties are sufficiently uncertain equilibria exist in two-party-games of electoral competition when multiple issues are at stake. What is more, in these equilibria both parties announce issue by issue the policy preferred by a "median voter" that they consider to be relevant for the dimension. So this theory can be used to justify the common practice to look at separate issues when modelling democratic processes.

The power of the uncertainty aversion assumption lies in the fact that the two parties face no uncertainty about the decisions of voters when they both announce the same platform. The choice problem of the parties in our model is actually remarkably similar to the choice problem of the individuals in the famous Ellsberg game (Ellsberg 1961). Just like the parties in our game of electoral competition the individuals in the Ellsberg game have to chose between a bet with a known 50% chance of winning and an alternative bet with unclear chances of winning. Experimental studies of this game consistently find that individuals, in violation of expected utility maximization, strictly prefer the bet with given odds, they exhibit uncertainty aversion. Consequently it should be possible to obtain a solution to the non-existence problem in Downsian voting with multiple issues using any of the decision theoretic models that rationalize the Ellsberg paradox.

In this paper we followed the approach of Gilboa and Schmeidler (1989) to establish a range of different sufficient conditions for the existence of equilibrium in various types of games of electoral competition. In particular we showed that games of electoral competition involving two issues have equilibria if parties are uncertain about the marginal rates of substitution. We showed furthermore that three dimensional games of electoral competition have equilibria if parties are in addition uncertain whether the electorate leans to the left or right.

The conditions for the existence of equilibrium given in the theorems of this paper are sufficient but not necessary. It is hoped that in the future some weaker conditions for the existence of equilibria will be established. A promising venue could be to assume Euclidean preferences and to restrict the set of permissible voter ideal point distributions and then ask what amount of uncertainty is sufficient to establish that equilibria exist. In particular we hope to show that under the assumption that the society is not polarized (following Caplin and Nalebuff (1988)) a small amount of uncertainty on voter ideal point distribution is sufficient to establish the existence of equilibria in $d-$dimensional games.

Another extension of this research could be to apply the same model of uncertainty aversion to solve different but related non-existence problems in political economy. Some models of ideologically motivated parties for example are plagued by similar non-existence problems as the Downs model. A main challenge in extending the present framework to such models lies in defining the utilities of parties. Political competition with more than 2 parties could be another area of application. Platform positioning models with more than two parties generally lack equilibria. Uncertainty aversion could be one of the modelling approaches to trim the model's incentives to deviate down to a more realistic lower level.

Furthermore, in a companion paper (Bade 2004) we show that with a different approach towards modelling uncertainty averse actors (following Bewley 2002), there might be equilibria in which office motivated parties announce different platforms. Our explanation for platform divergence does not need any *ad hoc* assumptions on the ideological motivation or parties, the driving force of this result are non-convexities in the preferences of voters.

7 Appendix

Proof of Proposition 1: We will need the following intermediate result.

Define $\psi' : \mathcal{B}(X \times H) \to [0,1]$ by $\psi'(A) := \int_{\mathbb{P}^*(X \times H)} \psi(A)p(d\psi)$, for an arbitrarily fixed $p \in \mathcal{P}$. This is well-defined as $\psi \mapsto \psi(A)$ is a Borel-measurable function, which can be shown using Theorem 14.13 in Aliprantis and Border (1999), letting $f = 1_A$. Moreover, it is obvious that $\psi'(\varnothing) = 0$ and $\psi'(X \times H) = 1$. To conclude that ψ' is a Borel probability measure on $X \times H$, let $\{A_1, A_2, ...\}$ be a countable set of mutually disjoint Borel sets in $X \times H$, and notice that

$$\psi'\left(\bigcup_{i=1}^{\infty} A_i\right) = \int\limits_{\mathbb{P}^*(X \times H)} \psi\left(\bigcup_{i=1}^{\infty} A_i\right) p(d\psi) = \sum_{i=1}^{\infty} \int\limits_{\mathbb{P}^*(X \times H)} \psi(A_i)p(d\psi) = \sum_{i=1}^{\infty} \psi'(A_i).$$

by the monotone convergence theorem. Since it is also easy to show that ψ is non-atomic and ψ_a has full support, it follows that $\Psi \subseteq \mathbb{P}^*(X \times H)$.Moreover, for any $p, q \in \mathcal{P}$ and any $\lambda \in [0,1]$ we have

$$\lambda \int\limits_{\mathbb{P}^*(X \times H)} \psi p(d\psi) + (1-\lambda) \int\limits_{\mathbb{P}^*(X \times H)} \psi q(d\psi) = \int\limits_{\mathbb{P}^*(X \times H)} \psi \; (\lambda p + (1-\lambda)q)(d\psi),$$

so the convexity of Ψ follows from that of $\mathcal{P}$. Finally, for any $(x, y) \in X^2$, by the definition of π_ψ,

$$\begin{aligned}\int\limits_{\mathbb{P}^*(X\times H)} \pi_\psi(x,y)p(d(\psi)) &= \int\limits_{\mathbb{P}^*(X\times H)} \left(\psi\left(T(x,y)\right)+\frac{1}{2}\psi\left(I(x,y)\right)\right)p(d\psi) \\ &= \psi'\left(T(x,y)\right)+\frac{1}{2}\psi'\left(I(x,y)\right)=\pi_{\psi'}(x,y).\end{aligned}$$

This completes the proof of Proposition 1. ■

A finite Intuition for (A1)

Suppose X and H had only two elements each ($X=\{x_1,x_2\}, H=\{h_1,h_2\}$). As in the main text an electorate consits of a distribution $\psi\in\mathbb{P}(X\times H)$. A party's belief is an element of the set $\mathbb{P}(\mathbb{P}(X\times H)))$. Every electorate ψ can be represented as the product of a distribution of voter ideal points X, denoted by $\psi_x\in\mathbb{P}(X)$ the marginal distribution of ψ with respect to x and a conditional probability distribution $\psi_{h|x}: X\to H$ assigning every ideal point $x\in X$ a probability distribution over the types $h\in H$. The set of all possible conditional probability distributions can be identified with $[0,1]^2$ where the first component denotes the probability that a voter with ideal point x_1 is of type h_1 whereas the second component is defined as the probability that a voter with ideal point x_2 is of type h_1. So the set of pssobile electorates $\mathbb{P}(X\times H)$ can alternatively be represented as $\mathbb{P}(X)\times[0,1]^2$. In analgoy the set of beliefs on the electorate can be represented as $\mathcal{P}\subseteq\mathbb{P}(\mathbb{P}(X)\times[0,1]^2)$

Now assume that parties independently form their belief on $\mathbb{P}(X)$ and $[0,1]^2$, that is assume that there exist convex and compact subsets of $\mathcal{P}_x\subseteq\mathbb{P}(\mathbb{P}(X))$ and $\mathcal{P}_h\subseteq\mathbb{P}([0,1]^2)$ such that $\mathcal{P}=\mathcal{P}_x\times\mathcal{P}_h$. We have shown for the more general context that Ψ the set of all expected electorates with respect to priors in $\mathcal{P}$ is a subset of $\mathbb{P}(X\times H)$ (Proposition 1.). In this more special context it is easy to see that for a set of priors that is decomposable as given above ($\mathcal{P}=\mathcal{P}_x\times\mathcal{P}_h$) , we have that there exist $\Psi_x\subset\mathbb{P}(X)$ and $\Psi_g\subset[0,1]^2$ such that $\Psi=\Psi_x\times\Psi_g$.

In short: if there where only two voters ideal points and two shapes to consider we could derive (A1) from an assumption on the formation of party beliefs. This logic can easily extend to any finite number of voter ideal points and preferences shapes. In any of these cases the conditional probabilities $\psi_{h|x}$ can be characterized by a finite set of numbers. However, it is anything but easy to extend this logic to the case considered in this paper: $X\subseteq\mathbb{R}^d$ a continuum. In this case the conditional probabilities $\psi_{h|x}$ belong to an infinite dimensional space. It is not clear how a probability should be defiend on this space. We therefore chose to simply impose (A1) on the set of party beliefs.

Proof of Theorem 1:

Lemma 2 (Rae and Taylor, 1971): *Any game of the form* $(2, X, \{\mu\}, \{\alpha\})$ *has an equilibrium.*

Proof: Suppose some profitable deviation $y \gg \mathbf{0}$ existed (remember that we normalized $\mathbf{m}(\mu) = \mathbf{0}$). Let A be the set of voters that are indifferent between $\mathbf{0}$ and y. Given that every voter has the same shape α we have that all voters $a' \notin A$ for which there exists an $a \in A$ such that $a' \ll a$ strictly prefer $\mathbf{0}$ to y. If $A \cap \{a|a_1 = 0\} = \varnothing$ or $A \cap \{a|a_2 = 0\} = \varnothing$ then since the A is a connected set either all voters in $\{a|a_1 \leq 0\}$ or all voters in $\{a|a_2 \leq 0\}$ will prefer $\mathbf{0}$ to y. Figures 1 and 2 give two examples for these two cases; the dashed lines represent the set A, observe that in either there is one axis that does not intersect with the dashed lines. Since $(\mathbf{0}, \mathbf{0})$ is the median vector we have $\mu(\{a|a_1 \leq 0\}) \geq \frac{1}{2}$ and $\mu(\{a|a_2 \leq 0\}) \geq \frac{1}{2}$ so in either case at least half the electorate votes for $\mathbf{0}$ and therefore such a deviation to y cannot raise the deviating party's vote share.

Let us now consider the remaining case in which $A \cap \{a|a_1 = 0\} \neq \varnothing$ and $A \cap \{a|a_2 = 0\} \neq \varnothing$. This only holds for deviations y such that $y_1\alpha_1 = y_2\alpha_2$. In this case all voters in $\{a|a_1 \leq 0 \text{ and } a_2 \geq y_2\}$ and in $\{a|a_1 \geq y_1 \text{ and } a_2 \leq 0\}$ are indifferent between $\mathbf{0}$ and y. Since we assume that all indifferent voters vote for either platform with equal probability we only need to look at the voters that strictly prefer one platform to the other. The set of voters strictly preferring y to $\mathbf{0}$ is a subset of $\{a|a_1 > 0, a_2 > 0\}$ whereas the set of voters that strictly prefer $\mathbf{0}$ to y is a superset of $\{a|a_1 < 0, a_2 < 0\}$. But since $(\mathbf{0}, \mathbf{0})$ is the median vector of the non-atomic μ we have $\mu(\{a|a_1 > 0, a_2 > 0\}) = \mu(\{a|a_1 < 0, a_2 < 0\})$. Consequently it cannot be that such a deviation increases the vote share. But by the same arguments no other deviation y raises the vote share to the deviating party and $(\mathbf{0}, \mathbf{0})$ is a political equilibrium. $\blacksquare$.

Lemma 3: *Let* $d \in \mathbb{N}$, *and let* $(d, X, \{\mu\}, G)$ *be a game in which parties are uncertain about the marginal rates of substitution. Then, for all* $y \in X$, *there exists an electorate* ψ^y *in* $\Psi(\{\mu\}, G)$ *such that* $\pi_{\psi^y}(y, \mathbf{0}) \leq \pi_{\mu \times \delta_\alpha}(y, \mathbf{0})$.

Proof: Fix an arbitrary $y \in X$. Since parties are uncertain about the marginal rates of substitution, there exists an $\alpha \in \mathbb{R}^d_{++}$ such that in all G_i there are some differentiable functions h_i and k_i with $h_i' \geq \alpha_i \geq k_i'$. Define the map $a \longmapsto (g_1^a, ..., g_d^a) =: g^a$ by

$$g_i^a := \begin{cases} h_i, & \text{if } |y_i - a_i| \geq |a_i| \\ k_i, & \text{otherwise} \end{cases}, \; i = 1, ..., d.$$

We begin by showing that $u_a^\alpha(\mathbf{0}) > u_a^\alpha(y)$ implies $u_a^{g^a}(\mathbf{0}) > u_a^{g^a}(y)$ for all a. Observe, first of all, that $u_a^\alpha(\mathbf{0}) > u_a^\alpha(y)$ implies

$$\sum_{i\in H}\alpha_i(|y_i-a_i|-|a_i|) > \sum_{i\notin H}\alpha_i(|a_i|-|y_i-a_i|),$$

where $H := \{i \in \{1,...,d\}|\ |y_i-a_i| \geq |a_i|\}$.Then, since $h_i' \geq \alpha_i \geq k_i'$,

$$\begin{cases} h_i(|y_i-a_i|)-h_i(|a_i|) \geq \alpha_i|y_i-a_i|-\alpha_i|a_i|, & \text{if } |y_i-a_i| \geq |a_i| \\ \alpha_i|a_i|-\alpha_i|y_i-a_i| \geq k_i(|a_i|)-k_i(|y_i-a_i|), & \text{otherwise.} \end{cases}$$

Combining these inequalities with the definition of g^a we obtain

$$\sum_{i=1}^{d}(g_i^a(|y_i-a_i|)-g_i^a(|a_i|)) > 0,$$

that is, $u_a^{g^a}(\mathbf{0}) > u_a^{g^a}(y)$. One can show similarly that $u_a^\alpha(\mathbf{0}) = u_a^\alpha(y)$ implies $u_a^{g^a}(\mathbf{0}) \geq u_a^{g^a}(y)$ for all a.

Next define an electorate ψ^y such that on the one hand the conditional distribution of shapes given a voter ideal point a can be described by the map $a \mapsto g^a$, that is we assume that $\psi(g^a|a) = 1$ for all a and that on the other hand the voter ideal point distribution ψ_a equals μ. Since the map $a \mapsto g^a$ is measurable, ψ^y is well-defined. Moreover, by construction, we have that $\psi^y \in \Psi(\{\mu\}, G)$. The construction of the map $a \longmapsto g^a$ also implies:

$$\begin{aligned}
\pi_{\psi^y}(y,\mathbf{0}) &= 1-\psi^y\{(a,g)|u_a^g(\mathbf{0}) > u_a^g(y)\}-\frac{1}{2}\psi^y\{(a,g)|u_a^g(\mathbf{0}) = u_a^g(y)\} \\
&= 1-\mu\left\{a|u_a^{g^a}(\mathbf{0}) > u_a^{g^a}(y)\right\}-\frac{1}{2}\mu\left\{a|u_a^{g^a}(\mathbf{0}) = u_a^{g^a}(y)\right\} \\
&\leq 1-\mu\{a|u_a^\alpha(\mathbf{0}) > u_a^\alpha(y)\}-\frac{1}{2}\mu\{a|u_a^\alpha(\mathbf{0}) = u_a^\alpha(y)\} \\
&= 1-(\mu\times\delta_\alpha)\{(a,g)|u_a^g(\mathbf{0}) > u_a^g(y)\})-\frac{1}{2}(\mu\times\delta_\alpha)\{(a,g)|u_a^g(\mathbf{0}) = u_a^g(y)\} \\
&= \pi_{\mu\times\delta_\alpha}(y,\mathbf{0}),
\end{aligned}$$

and we are done. ■

Proof of Theorem 1: By Proposition 1 we know that the only candidate for an equilibrium is $(\mathbf{0},\mathbf{0})$. The payoff to a party that deviates to y is $\min_{\psi\in\Psi(\{\mu\},G)}\pi_\psi(y,\mathbf{0})$. By Lemma 3, $\pi_{\mu\times\delta_\alpha}(y,\mathbf{0})$ is an upper bound on this payoff. Finally by Lemma 2, according to the electorate $\mu\times\delta_\alpha$ no profitable deviation form $(\mathbf{0},\mathbf{0})$ exists. Thus,

$$\min_{\psi\in\Psi(\{\mu\},G)}\pi_\psi(y,\mathbf{0}) \leq \pi_{\psi^y}(y,\mathbf{0}) \leq \pi_{\mu\times\delta_\alpha}(y,\mathbf{0}) \leq \frac{1}{2} = \min_{\psi\in\Psi(\{\mu\},G)}\pi_\psi(\mathbf{0},\mathbf{0})$$

where ψ^y is the electorate constructed in Lemma 3. So there is no profitable deviation for either party, and $(\mathbf{0}, \mathbf{0})$ is an equilibrium of $(2, X, \{\mu\}, G)$. ■

The following Lemma 4 is a major building block of the proof of Theorem 2.

Lemma 4: *A game of the form* $(3, X, \{\mu\}, \{\alpha\})$ *has an equilibrium if and only if* μ *is balanced.*

Proof: We define the function $\mathbf{sgn}$: $\mathbb{R}^3 \to \{1, 0, -1\}^3$ by $\mathbf{sgn}(x) = (\text{sgn}(x_1),\text{sgn}(x_2),\text{sgn}(x_d))$, and let $A_f^\mu := \{x|\mathbf{sgn}(x - m(\mu)) = f\}$ for all $f \in \{1, -1\}^3$. As Lemma 4 covers games with certainty we can, for ease of exposition, revert to the normalization $\mathbf{m}(\mu) = \mathbf{0}$. We therefore also drop μ from the notation of these sets and now write A_l, A_r and A_f.

We first show that $\mu(A_l) = \mu(A_r)$ if and only if $\mu(A_f) = \mu(A_{-f})$ for all f, then we show that this condition is necessary and sufficient for $(\mathbf{0}, \mathbf{0})$ being an equilibrium.

Let $\mu(A_l) = \mu(A_r)$. Define 4 variables $D_f := \mu(A_f) - \mu(A_{-f})$ for all f with $f_1 = 1$. Since $\mathbf{0}$ is the median vector of μ we have that $\sum\limits_{f_i=1} D_f = 0$ for all $i = 1, 2, 3$. Given $D_{(1,1,1)} = \mu(A_r) - \mu(A_l) = 0$ this reduces to a system of 3 linearly independent equations in 3 unknowns, the only solution is $D_f = \mu(A_f) - \mu(A_{-f}) = 0$ for all f.

Given $\mu(A_f) = \mu(A_{-f})$ for all f we show that for any deviation from $(\mathbf{0}, \mathbf{0})$ the party remaining at $\mathbf{0}$ gets at least half the vote share. First we derive a condition under which all voters in some A_f vote for $\mathbf{0}$. Then we use this condition to show that given the choice between the platforms $y \neq \mathbf{0}$ and $\mathbf{0}$ for any f either all voters in A_f or all voters in A_{-f} vote for $\mathbf{0}$ or no voter in either A_f or A_{-f} strictly prefers y to $\mathbf{0}$.

All voters in A_f vote for $\mathbf{0}$ if for all $a \in A_f$ the utility from platform $\mathbf{0} : -\sum_{i=1}^3 \alpha_i|a_i|$ is larger than the utility from the other platform: $-\sum_{i=1}^3 \alpha_i|y_i - a_i|$. So all voters in A_f strictly prefer $\mathbf{0}$ to y if

$$\sup_{a\in A_f}\left(\sum_{i=1}^3 \alpha_i|a_i| - \sum_{i=1}^3 \alpha_i|y_i - a_i|\right) < 0.$$

Considering all possible cases, we can solve for the absolute value of $|a_i|$ and $|y_i - a_i|$ and receive

$$\sup_{a\in A_f}\left(\sum_{y_i\geq 0\geq a_i}(-\alpha_i y_i) + \sum_{0\leq a_i\leq y_i}(2\alpha_i a_i - \alpha_i y_i) + \sum_{0\leq y_i\leq a_i}\alpha_i y_i + \right.$$
$$\left. + \sum_{y_i<0<a_i}\alpha_i y_i + \sum_{y_i\leq a_i<0}(-2\alpha_i a_i + \alpha_i y_i) + \sum_{a_i<y_i\leq 0} - \alpha_i y_i\right) = \sum_{i=1}^3 f_i\alpha_i y_i.$$

So if this expression is negative we are done all voters in A_f vote for $\mathbf{0}$. If this expression is positive we have

$$-\sum_{i=1}^{3} f_i \alpha_i y_i = \sum_{i=1}^{3} - f_i \alpha_i y_i < 0$$

and therefore all voters in A_{-f} vote for $\mathbf{0}$. Finally if $\sum_{i=1}^{d} f_i \alpha_i y_i = 0$ then

$$\sup_{a \in A_f} \left(\sum_{i=1}^{3} \alpha_i |a_i| - \sum_{i=1}^{3} \alpha_i |y_i - a_i| \right) = \sup_{a \in A_{-f}} \left(\sum_{i=1}^{3} \alpha_i |a_i| - \sum_{i=1}^{3} \alpha_i |y_i - a_i| \right) = 0.$$

and consequently no voter in either A_f or A_{-f} strictly prefers platform y.

Given that $\mu(A_{-f}) = \mu(A_f)$ for all f we can now show that $\sum_{f_1=1} \mu(A_f)$ represents a lower bound on $\pi(\mathbf{0}, y)$. This is so since, on the one hand, for any f with $f_1 = 1$ and any deviation y either all voters with ideal point in A_f or all voters with ideal point in A_{-f} or one half of the voters with ideal point in $A_f \cup A_{-f}$ vote for $\mathbf{0}$, and, on the other hand, $\mu(A_{-f}) = \mu(A_f)$ for all f. Finally since $\mathbf{0}$ is the median vector of μ we know that $\sum_{f_1=1} \mu(A_f) = \frac{1}{2}$ and we have $\pi_\mu(\mathbf{0}, y) \geq \frac{1}{2}$ and therefore no deviation from $(\mathbf{0}, \mathbf{0})$ that raises the vote share of the deviating party exists.

Now suppose μ where not balanced, that is assume $\mu(A_l) = p$ and $\mu(A_r) = q$ with $p > q$. Let the values of all $\mu(A_f)$ be given by the following chart:

f	$\mu(A_f)$
$(1,1,1)$	q
$(1,1,-1)$	r
$(1,-1,1)$	s
$(1,-1,-1)$	$\frac{1}{2} - s - r - q$
$(-1,-1,-1)$	p
$(-1,-1,1)$	$s + (q - p)$
$(-1,1,-1)$	$r + (q - p)$
$(-1,1,1)$	$\frac{1}{2} - s - r - q - (q - p)$

When a deviator plays $\lambda(\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \frac{1}{\alpha_3})$ with $\lambda > 0$ against $\mathbf{0}$, then all voters in A_l, $A_{(1,-1,-1)}$, $A_{(-1,-1,1)}$ and $A_{(-1,1,-1)}$ are voting for $\mathbf{0}$. In the limit for $\lambda \to 0$ only these voters will vote for $\mathbf{0}$. So in the limit the vote share of the remaining party is $\frac{1}{2} + q - p$. Since the vote share of the remaining party decreases continuously with λ there exists some $\lambda^* > 0$ such that $\pi(\lambda^*(\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \frac{1}{\alpha_3}), \mathbf{0}) > \frac{1}{2}$ and $(\mathbf{0}, \mathbf{0})$ cannot be an equilibrium. ■

Before proceeding with the proof of Theorem 2 let us remark that Lemma 4 generalizes Lemma 3 as any 2-dimensional distribution of voter ideal points is balanced. Secondly, observe that balancedness of μ does not imply $\mu(A_f) = \mu(A_{-f})$ for all f, for higher dimensional issue spaces.

Proof of Theorem 2: We start by showing that there exists a balanced μ in Λ. By our assumption that the parties are uncertain as to whether the electorate leans to the left or right, there exists a left leaning μ_l and a right leaning μ_r in Λ. For all $\lambda \in [0,1]$ define $\mu_\lambda = \lambda\mu_l + (1-\lambda)\mu_r$. Define

$$\begin{array}{rcl} f & : & [0,1] \to [-1,1] \\ f(\lambda) & = & \mu_\lambda(A_l^{\mu_\lambda}) - \mu_\lambda(A_r^{\mu_\lambda}) \end{array}$$

a continuous function. Clearly: $f(1) > 0$ and $f(0) < 0$ so there exists some $\lambda^b \in (0,1)$ such that $f(\lambda^b) = 0$. Observe that μ_{λ^b} is balanced. Since Λ convex we also have that $\mu_{\lambda^b} \in \Lambda$.

By the same argument as forwarded in the proof of Theorem 1 we know that $(\mathbf{m}(\mu_{\lambda^b}), \mathbf{m}(\mu_{\lambda^b}))$ is an element of $\mathcal{E}(3, X, \{\mu_{\lambda^b}\}, G)$. Finally

$$\min_{\psi\in\Psi(\Lambda,G)} \pi_\psi(y, \mathbf{m}(\mu_{\lambda^b})) \leq \min_{\psi\in\Psi(\{\mu_{\lambda^b}\},G)} \pi_\psi(y, \mathbf{m}(\mu_{\lambda^b}))$$

and therefore $(\mathbf{m}(\mu_{\lambda^b}), \mathbf{m}(\mu_{\lambda^b}))$ is also an element of $\mathcal{E}(3, X, \Lambda, G)$. ■.

Proof of Theorem 3: Following the proof of Theorem 2, observe that once we had established that $\mu(A_f) = \mu(A_{-f})$ for all f in Lemma 3 we made no more use of either balancedness or 3-dimensionality in the proofs of Lemma 3. So we note the following Lemma 5 in passing:

Lemma 5: *Take an d-dimensional game* $(d, X, \{\mu\}, \{\alpha\})$ *with* $\mathbf{m}(\mu) = \mathbf{0}$. *This game has an equilibrium if* $\mu(A_f) = \mu(A_{-f})$ *for all* f .

The proof of Theorem 3 proceeds like that of Theorem 2 replacing Lemma 4 by Lemma 5. ■

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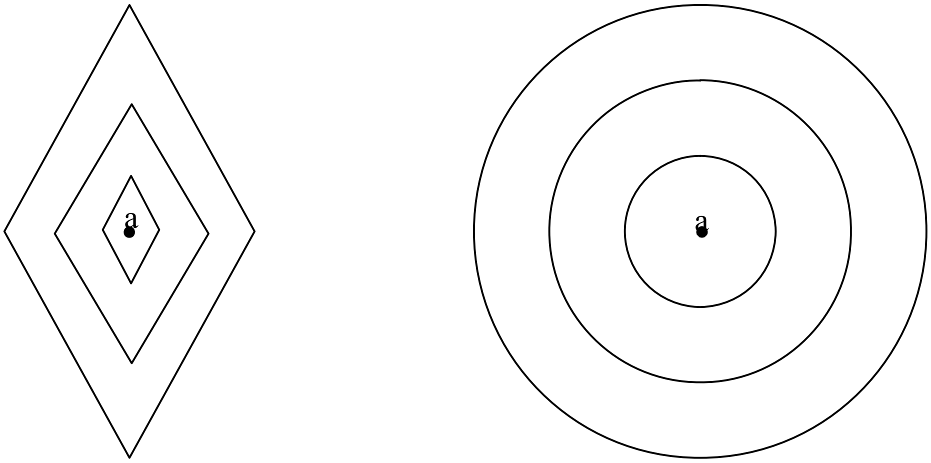


Indifference curves of a voter (a,α) Indifference curves of a voter (a,g^{o})

Figure 1:

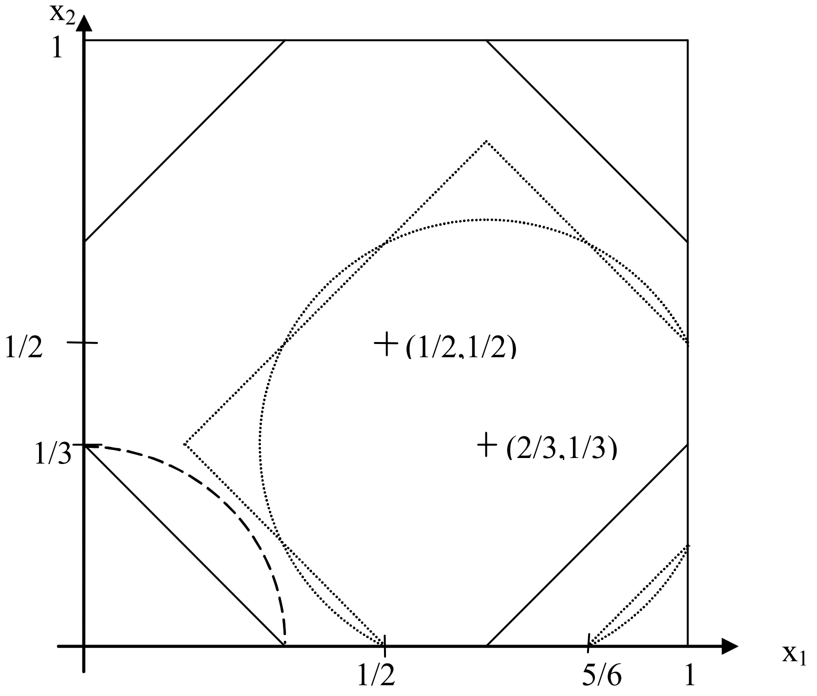
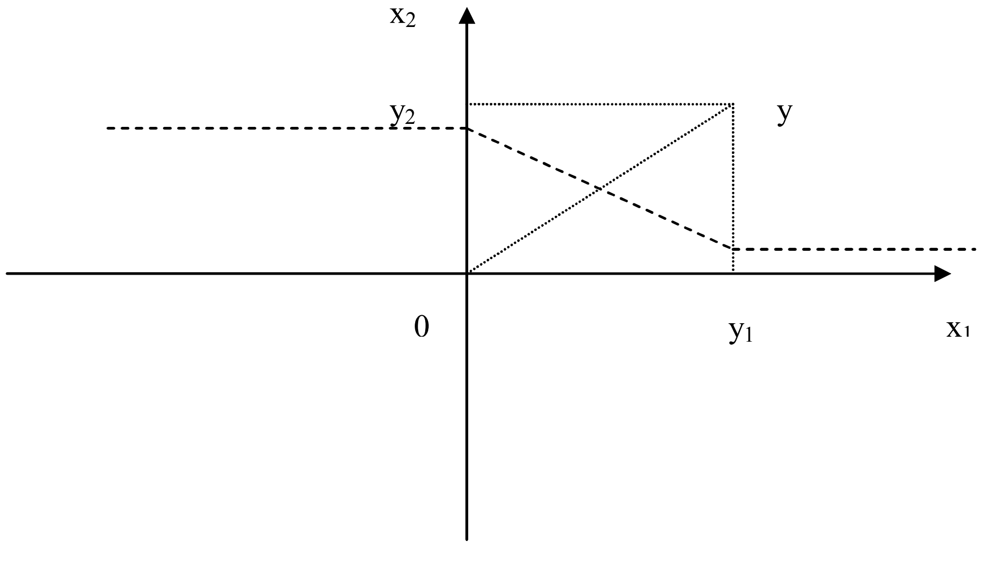


Figure 2:



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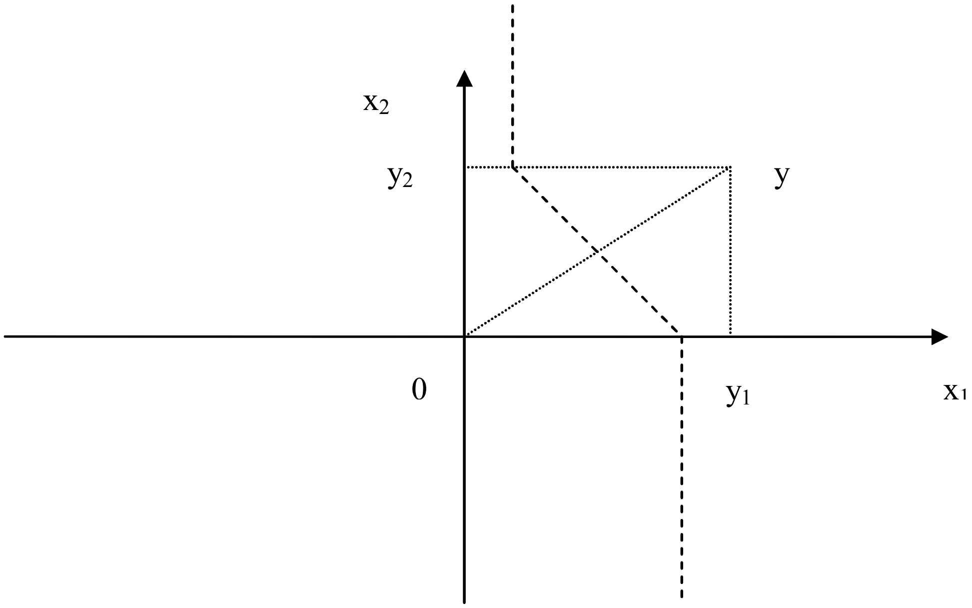

Figure 4:

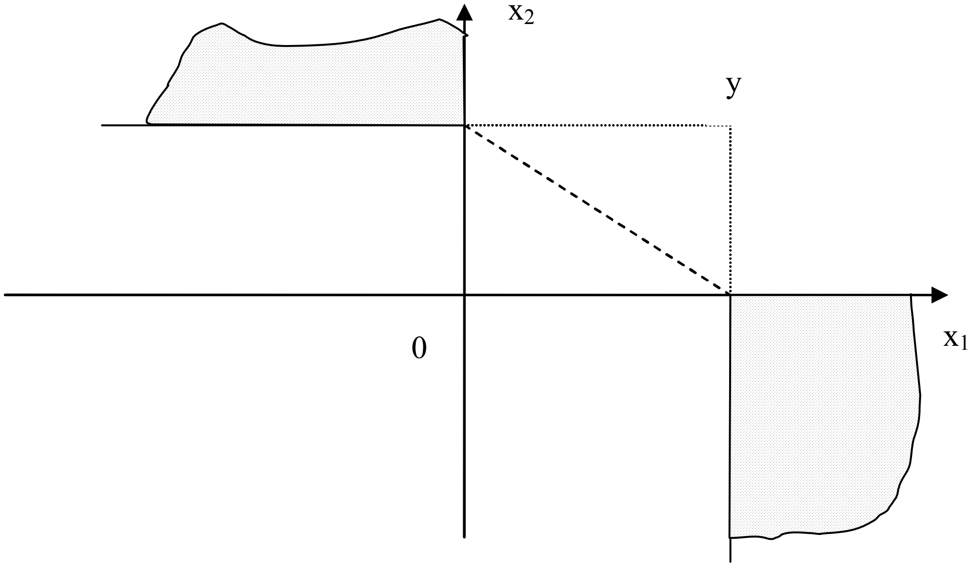

Figure 5: