Distributional effects of hiring through networks

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Abstract

We present a variant of Galenianos (2011), a version of a random search model with two matching technologies: a standard matching function and worker networks. Our model has two types of workers, networked workers and non-networked workers. A steady state equilibrium exists where networked workers have lower unemployment and higher wages, and it is unique under some conditions. Then we ask a question: how would a policy that bans the use of networks in hiring (e.g., anti-old boy network laws) affect welfare? It is shown that the effects of such a policy on non-networked workers can be either positive or negative, depending on model parameters. In our calibration, such a policy would make non-networked workers slightly worse off and networked workers substantially worse off.

Keywords: random search, network, referral, policy analysis, welfare, dynamics.

JEL classifications: C78, E24, E60, I3, J20, J30.

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1 Introduction

Suppose one is currently unemployed while some of his relatives and friends are employed. Then these people sometimes find him a job; they refer him to their employers. Such social networks also include so-called old-boy networks (i.e., alumni of the same school).[1] As Topa (2010) well summarizes, the use of social networks of workers in the labor market is widespread and seems important.[2] The role of networks is often associated with so-called 'search friction' that causes the co-existence of unemployment and vacancies. After all, there is a lot of unmodeled heterogeneity in workers' skills and specializations, kinds of jobs, locations, etc., so it takes time for workers and firms to be matched with a right partner. One important role of networks is that one's network friends know his qualities and hence can tell if he is suitable for the kind of job their employers have. This aspect of networks potentially mitigates the search friction and saves firms hiring cost. For example, some university departments only hire their own Ph.D's to get a qualified worker without reviewing application materials, having interviews, etc.

On the other hand, the use of social networks in hiring is sometimes restricted. Many govermental/public institutions attempt to avoid hiring through social networks and use more formal channels by posting job openings in public. Also, some policies simply restrict one from hiring a person who belongs to the same social group. (e.g., anti-nepotism laws for public jobs, affirmative actions regarding race, sex, nationality, caste, etc.). In some cases, avoiding informal channels is for the sake of diversity. In other cases, it is because hiring through informal channels is not regarded as fair for those without any such connections.

A natural question arises: what is the welfare implication of hiring through social net-

[1]In this paper, the word 'network' is used in a rather casual sense and does not mean graph-theoretic structures.

[2]A large proportion of workers attempt to use their personal networks in their job search alongside other methods (e.g., Holzer (1987), Elliott (1999)) and many of them actually find their jobs through networks (e.g., Granovetter (1995), Lin et al. (1981)). Also, many surveys report positive effects of the use of personal networks on wages (e.g., Korenman and Turner (1996)).

works? In particular, are workers without a network hurt because others are hired through networks? In other words, would they benefit from, for example, expanding anti-nepotism laws or affirmative actions to the entire economy? To answer this question, we study a version of the random search model with two permanent types of workers: workers with a social network and workers without one. The model also has two ways of filling jobs, a formal channel (a standard matching function for which firms pay hiring cost) and worker networks, and firms can use both methods. A steady state equilibrium exists where networked workers have lower unemployment and higher wages, and it is unique under some conditions.

Then we ask a question: how would a policy that bans the use of worker networks in hiring affect the welfare of workers? We compare the welfare of the pre-policy economy (the steady state with networks playing a role) and the welfare of the post-policy economy (the transition path to the new steady state). We show that whether the effect of such a policy on non-networked workers is positive or negative depends upon model parameters. In our calibration, such a policy would make non-networked workers slightly worse off and networked workers substantially worse off. This result may seem surprising if one imagines a situation where workers are competing over a fixed number of available posts. In reality, the number of posted vacancies is also determined in equilibrium, affected by the intensity of search friction. Our simple model suggests that the effectiveness of such a policy that increases search friction is ambiguous even for policy-makers who care about people without any networks or connections.

Labor-search models with 'network' components are divided into two types. On one hand, there are models with exogenous vacancies. For example, each agent finds a job opening with some exogenous propability and passes that job on to one of his friends if he is employed. Montgomery (1991, 1992, 1994), Calvo-Armengol and Jackson (2004, 2007), and Mayer (2011) belongs to this type. On the other hand, there are models with endogenous vacancies as Mortensen and Pissarides (1994); there is a matching function and the number of vacancies posted by firms is determined by firm free-entry condition. This class includes Calvo-Armengol and Zenou (2005), Fontaine (2008), Kuzubas (2010), and Galenianos (2011). Among them, Galenianos (2011) is different from others. In the first three, a network component is added to the matching function. A firm goes through costly labor search and is matched with a worker through the matching function, but this worker, if he is already employed, passes that job offer to one of his unemployed friends. Galenianos (2011), on the other hand, regards the matching function (i.e., costly search) and network matching as two distinct matching technologies. Firms can search for a worker through formal costly process. But at the same time, they can also count on their current employee to find a qualified person from among his network friends. This paper borrows Galenianos's network matching to address the welfare-related question. The model is however different from Galenianos in other respects.[3]

2 Model

The model is a variant of Galenianos (2011). Time is continuous and the horizon is infinite.

Workers and firms

There are continuum-many workers and their total measure is one. Each worker has an exogenously given type $j \in \{0, 1\}$. Type-0 workers have no networks ('non-networked workers') and type-1 workers have networks ('networked workers'). These two types are identical in the other respects. In particular, they have the same productivity. Let n_0 and n_1 be the proportions of non-networked and networked workers, respectively. It follows that $n_0 + n_1 = 1$. There are also continuum-many firms and their total measure is infinite. Both workers and firms are risk-neutral, maximizing expected discounted income with discount factor r.

[3]In particular, Galenianos does not allow workers to have no networks at all.

At any point in time, each worker is either employed or unemployed. If he is unemployed, he gets flow output $b > 0$ from home production. Each firm can employ at most one worker, so we will use the terms *firm* and *job* interchangeably.[4] Each firm/job is either incumbent (i.e., filled with a type-j worker) or unmatched. Incumbent firms produce constant flow output $y > b$. Each unmatched firm is either posting or non-posting. Posting firms pay flow cost k to keep posting its vacancy.[5] Entry is costless so firms currently not posting a vacancy can get posting status simply by starting to pay the flow cost.

Matching technologies

New matches between workers and firms are created in two ways and firms can use both. Existing matches are destroyed at some exogenous rate δ, in which case the worker becomes unemployed and the firm becomes unmatched.

One matching technology is described by the following version of the standard matching function. Let u_j be the unemployment rate of type-j workers and $u \equiv n_0 u_0 + n_1 u_1$ be the total unemployment rate. Furthermore, let v be the measure of vacant firms. Then the matching function creates a flow $M(u, v) \equiv \mu u^{1-\eta} v^{\eta}$ of matches between u unemployed workers and v vacant firms, where $\mu > 0$ and $\eta \in (0, 1)$. Therefore, for each unemployed worker, the Poisson arrival rate of a job offer through the matching function is $M(u, v)/u = \mu\theta^{\eta}$, where $\theta \equiv v/u$ is the labor market tightness. For each vacant firm, the arrival rate of a type j-worker is $M(u, v)/v \times (n_j u_j/u) = \mu\theta^{\eta-1} \times (n_j u_j/u)$.

The other matching technology uses in-network referrals. All firms know the same large number of other firms and all type-1 workers know the same large number of other type-1 workers. An existing match between an incumbent firm (firm A) and a type-1 employee (worker A) generates another potential match with Poisson rate $\rho \geq 0$. The timeline is as

[4] One job per firm is a common assumption and it is innocuous under the constant returns to scale production and cost of hiring. We will come back to this point in Section 5.

[5] The interpretation of this cost includes cost to post newspaper ads, review application materials, have interviews, etc.

follows. Firm A knows another unmatched firm (firm B) and worker A knows a friend from his network (worker B), and their finding is that firm B and worker B could potentially be a good match. In such an event, worker A contacts worker B. If worker B is currently unemployed, which will be the case with probability u_1, worker A refers her to his employer (firm A). Then firm A refers worker B to the unmatched firm B and a new match is created.Therefore, from the incumbent firm A's point of view, the arrival rate of network matches is ρu_1. From the point of view of the unemployed type-1 worker B, the arrival rate of referrals is $(1-u_1)\rho$. That is, the arrival of referrals is assumed to depend upon the employment rate of their network friends because only existing matches generate referrals.

An important feature of the network matching is that firms do not have to pay hiring cost k to get a worker referred to them. Any unmatched firm, whether it is currently posting a vacancy or not, could potentially get a referral. Hence each unmatched firm's decision is whether they should just wait for referrals or they should also post a vacancy while waiting for referrals.

Wages and values

A firm that employs a type-j worker pays flow wage w_j to its employee. A type-j worker has lifetime value W_j if he is employed and U_j if he is unemployed. Also, let the lifetime value of a firm be J_j if it is filled by a type-j worker and V if it is posting. When a filled firm refers its employee's friend to an unmatched firm and a new match is created, it receives the whole value $J_1 - V$ created by the new match. This is an implicit one-time payment from the unmatched firm to the incumbent firm.[6]

There is a set of Bellman equations associated with these values. The wage w_j is determined by the traditional Nash bargaining assumption, where the worker's bargaining power is $\beta \in (0, 1)$. We assume the free-entry condition $V = 0$ so a non-posting firm does not

[6]In Section 5, we show that one-firm-one-worker model with this referral mechanism is equivalent to the model in which firms can employ multiple workers.

gain positive net profit from getting a posting status by starting to pay hiring cost. That is, posting firms and non-posting firms both have value V.

Definition 1. *A* steady state equilibrium *is* $(U_i, W_i, J_i, w_i, u_i)_{i=0,1}$ *and* (V, v) *that satisfy*

$$
\begin{aligned}
rU_0 &= b + \mu\theta^\eta (W_0 - U_0) && (1)\\
rU_1 &= b + \mu\theta^\eta (W_1 - U_1) + \rho(1-u_1)(W_1 - U_1) && (2)\\
rW_0 &= w_0 + \delta(U_0 - W_0) && (3)\\
rW_1 &= w_1 + \delta(U_1 - W_1) && (4)\\
rJ_0 &= y - w_0 + \delta(V - J_0) && (5)\\
rJ_1 &= y - w_1 + \delta(V - J_1) + \rho u_1 (J_1 - V) && (6)\\
rV &= -k + \sum_{j=0,1} \mu\theta^{\eta-1} \frac{n_j u_j}{u} (J_j - V) && (7)\\
(W_j - U_j)/\beta &= (J_j - V)/(1-\beta), \quad j = 0, 1 && (8)\\
(1-u_0)\delta &= u_0 \mu\theta^\eta && (9)\\
(1-u_1)\delta &= u_1(\mu\theta^\eta + \rho(1-u_1)) && (10)\\
V &= 0, && (11)
\end{aligned}
$$

where $u = n_0 u_0 + n_1 u_1$ *and* $\theta = v/u$.

(1)–(7) are Bellman equations, (8) is wage determination, (9)–(10) are inflow-equal-outflow conditions, and (11) is the free-entry condition. That is, the equilibrium is defined as a solution to the system of 12 equations in 12 unknowns.

What makes this model different from the standard Mortensen-Pissarides model is the three terms involving ρ; the economy has one more matching technology (i.e., the last term of (10)), some workers have a chance to be hired through networks (i.e., the last term of (2)), and some incumbent firms have a chance to get profits from the use of their employee's networks (i.e., the last term of (6)).

Proposition 1. *There exists a steady state equilibrium.*

The proof of the existence of a steady state is much harder than that of the Mortensen-Pissarides model due to the fact that the right hand sides of (2) and (6) include u_1, not just θ. The proof is one-by-one elimination of variables and found in the appendix. The following features are true in any steady state equilibrium:

Proposition 2. *(Dominance of networked workers)*
$u_0 > u_1$*,* $W_1 > W_0$*,* $U_1 > U_0$*, and* $w_1 > w_0$*.*

The higher wage for type-1 workers arises due to two reasons. First, the firm can potentially benefit from having an employee who sometimes refers a friend. So the firm compensates for that benefit by paying a higher wage to networked workers. Second, the networked worker has higher unemployment value because of the prospect of getting hired through his network. Therefore, he has higher reservation value in Nash bargaining over wage.

On the other hand, depending on the parameters, it is not necessarily the case that $J_1 > J_0$ in equilibrium. Indeed, it is quite possible that $J_0 > J_1$. In such a case, the interpretation is that the equilibrium wage for networked workers is so high that the firm prefers being matched with a non-networked worker. However, in any equilibrium, both J_0 and J_1 are higher than V, so posting firms hire whoever arrives first rather than remaining vacant and waiting for the next arrival of a different type of worker.

Next we give some results on comparative statics and uniqueness of equilibrium. For that purpose, consider the standard Mortensen-Pissarides economy, which coincides with our model when $\rho = 0$. That is, networks do not play a role, thereby rendering the differences between types virtually meaningless. We denote all the endogenous variables of the MP economy with subscript "m". For example, u_m is the (total) unemployment rate of the MP steady state, $\theta_m \equiv v_m/u_m$ is the market tightness of the MP steady state, etc. The unique existence of the MP steady state is easy to show.

Lemma 1. *The MP steady state* $(U_m, W_m, J_m, V_m, w_m, u_m, v_m, \theta_m)$ *exists and is unique.*

Due to Lemma 1, u_m is an implicit function of parameters $(r, \delta, \mu, \eta, y, b, k, \beta)$ that is determined outside our model. In this sense, u_m can be treated as exogenous to our model, although it is endogenous to the MP model.

Proposition 3. *(Dependence of steady states on* ρ*)*

In any equilibrium, u_1 *is decreasing in* ρ *near* $\rho = 0$*. Moreover,*

(i) if $u_m < \beta$*, then* u_0*,* J_0 *are increasing, and* v*,* θ*,* J_1*,* w_0*,* W_0*,* U_0 *are decreasing in* ρ *near* $\rho = 0$*; and*

(ii) if $u_m > \beta$*, then* u_0*,* u*,* J_0 *are decreasing, and* θ*,* J_1*,* w_0*,* W_0*,* U_0*,* w_1*,* W_1*,* U_1 *are increasing in* ρ *near* $\rho = 0$*.*

The effect of positive ρ on the economy is a compound of three effects, corresponding to the three terms involving ρ, each in (2), (6) and (10). When ρ is positive as opposed to zero, the economy has one more matching technology (the last term in (10)), so the total unemployment rate u is different. In addition, the term in (2) and the term in (6) imply two offsetting effects on firms. On one hand, positive ρ gives type-1 workers higher outside value during wage negotiations, which leads to higher wages for them and lower profits for firms. On the other hand positive ρ allows incumbent firms to potentially receive profits from network matchings. In case (i), the former dominates, discouraging firms' entry and decreasing θ. In turn, type-0 workers are disadvantaged by the existence of networked people. In case (ii), the latter dominates, increasing θ. In turn, type-0 workers actually benefit from the existence of networked people. As is shown after the proof of Lemma 1, u_m is increasing in β, k and b, and satisfies $\lim_{k\to\infty} u_m = 1$, $\lim_{\beta\to 0} u_m > 0$, etc. So both $u_m < \beta$ and $u_m > \beta$ are possible.

The following uniqueness property is useful when we compute the equilibrium in the next section.

Proposition 4. *(Uniqueness)*

Suppose $\eta \leq 0.5$*. Then,*

(i) at most one steady state equilibrium satisfies $u_1 \leq 0.5$*;*

(ii) $k \approx 0$ *is sufficient for uniqueness; and*

(iii) $\rho \approx 0$ *and* $u_m \leq 0.5$ *are sufficient for uniqueness.*[7]

3 Policy Implications

In this section, we study implications of the policy that bans hiring through networks, imposing $\rho = 0$. After such a policy is imposed, there is no longer a difference between type-0 and type-1 workers so the environment becomes that of Mortensen-Pissarides. (Hereafter, we mean $\rho = 0$ by "the MP model".) As the next proposition states, the MP model has very simple dynamics.

Proposition 5. *(Stability of the MP steady state)*

Consider the dynamics of the MP equilibrium:

$$\dot{U_m} = rU_m(t) - b - \mu\theta_m(t)^\eta[W_m(t) - U_m(t)] \tag{12}$$

$$\dot{W_m} = rW_m(t) - w_m(t) - \delta[U_m(t) - W_m(t)] \tag{13}$$

$$\dot{J_m} = rJ_m(t) - (y - w_m(t)) - \delta[V_m(t) - J_m(t)] \tag{14}$$

$$\dot{V_m} = rV_m(t) + k - \mu\theta_m(t)^{\eta-1}[J_m(t) - V_m(t)] \tag{15}$$

$$\dot{u_m} = (1 - u_m(t))\delta - u_m(t)\mu\theta_m(t)^\eta. \tag{16}$$

with Nash bargaining condition $(W_m(t) - U_m(t))/\beta = (J_m(t) - V_m(t))/(1-\beta)$, $\forall t$*, and the free-entry* $V_m(t) = \dot{V_m}(t) = 0$, $\forall t$*. Given the initial value for* u_m*, an equilibrium path is unique. Moreover,* $u_m(t)$ *converges to the steady state level gradually, while the wage, values,*

[7] $\eta \leq 0.5$ is not inconsistent with $u_m \leq 0.5$. For instance, Shimer (2005) calibrates η to be 0.28.

and $\theta_m(t)$ *are constant.*

When one studies policy effects, it is not legitimate to compare two steady states, one for $\rho > 0$ and the other for $\rho = 0$ (i.e., the MP steady state). The transition should be taken into account; we should compare the former steady state with a path that converges to the latter steady state after such a policy is imposed. The following is our scenario. Suppose that the economy starts with $\rho > 0$ and it is in the steady state with the total unemployment rate $u \equiv n_0 u_0 + n_1 u_1$. One day, say at time t_0, hiring through networks is banned by law in a permanent and unanticipated manner, so ρ is set to zero. Is such a policy good or bad? By the last proposition, while the economy's unemployment rate, starting with the initial level u, gradually converges to the MP steady-state level u_m, all the values and wages immediately jump to those of the MP steady state. Therefore, for values and hence welfares, the comparison in Proposition 3 is valid even though the whole economy doesn't immediately jump to the MP steady state. The result in Proposition 3, however, is limited to near $\rho = 0$, is only qualitative and still depends on parameters. Hence we perform the calibration below.

In the calibrated model, we first analyze how much each of the four groups of workers (type-0/type-1 and employed/unemployed) becomes better off (or worse off). In addition, we compute the total resource of the economy: match output plus home production minus vacancy cost. In the pre-policy steady state, the total resource is $[bu + y(1-u) - kv]/r$. In the post-policy economy, it is

$$Y_m(t_0) \equiv \int_{t_0}^{\infty} e^{-r(\tau - t)}[bu_m(\tau) + y(1 - u_m(\tau)) - kv_m(\tau)]d\tau, \text{ with } u_m(t_0) = u. \tag{17}$$

The following proposition is useful to compute it.

Proposition 6.

$$Y_m(t_0) = uU_m + (1-u)(W_m + J_m).$$

Table 1: Parameter calibration

t	the unit of time	quarter
r	interest rate	0.012 (Shimer)
δ	job destruction rate	0.1 (Shimer)
ρ	importance of networks	1.2
μ	coefficient of matching function	$0.45/0.63^{\eta}$
η	exponent of matching function	
y	output	1 (normalization)
b	home production	0.4 (Shimer)
k	vacancy cost	7.1
n_1	size of networked workers	0.85
β	worker's bargaining power	0.024

3.1 Calibration and numerical exercise

The unit of time is chosen as a quarter of a year. We have 10 parameters, summarized in Table 1. Match output y is normalized to one. We inherit from Shimer (2005) $r = 0.012$, $\delta = 0.1$, and $b = 0.4$. Regarding the size of the population of networked workers, Topa (2010) mentions, "Holzer (1987) uses data from the 1981-82 modules of the National Longitudinal Survey of Youth and finds that 87 % of currently employed and 85 % of currently unemployed workers used friends and relatives in their job search, alongside other methods." That is, approximately 85 % of workers have ever attempted to use networks in their job search along with other methods.[8] We interpret it as $n_1 = 0.85$.

There are five parameters left to be calibrated, k, μ, η, ρ and β. The following are our calibration targets.

1. $u = 5.67\%$ (U.S. average of 1951-2003, reported in Shimer (2005))

2. $\theta = 0.63$ (Hagedorn-Manovskii (2008))[9]

3. average wage $\frac{n_0(1-u_0)}{1-u}w_0 + \frac{n_1(1-u_1)}{1-u}w_1 = 0.666$ (Two thirds of output)

[8]Other methods include newspaper ads, direct contact, visits to state agencies, private agencies, school placement offices, etc.

[9]This number does not affect the result due to a well-known neutrality among v, k and μ.

Table 2: Computed welfare criteria

		Pre-policy	Post-policy	
			$\eta = 0.1$	$\eta = 0.4$
Unemployment rate	type-0	10.7%	5.67% (converge to 10.9 %)	5.67% (converge to 11.8%)
	type-1	4.78%		
Wage	type-0	0.502		
	type-1	0.693	0.501	0.494
Unemployed workers' value	type-0	0.490		
	type-1	0.678	0.489	0.482
Employed workers' value	type-0	0.491		
	type-1	0.679	0.490	0.483
Economy's total output		0.714	0.540	0.534

$\left(\text{Wage ratio} = \frac{w_1}{(n_0u_0/u)w_0+(n_1u_1/u)w_1} = 1.08 \right)$

4. Several surveys report that about 50 % of people find jobs through social contacts. In our model this implies

$$u\mu\theta^\eta \approx n_1u_1\rho(1-u_1)$$

Also Topa (2010) mentions, "Korenman and Turner (1996) also find that the use of social contacts increases wages by about 20 % in a survey of Boston youth, and by 7 % in a sample of young urban males from the 1982 NLSY." Its interpretation in our model is that the ratio of the wage of those hired through network matchings, w_1, to the average wage of those hired through the matching function, $(n_0u_0/u)w_0 + (n_1u_1/u)w_1$, is $1.07 - 1.20$. This can act as a barometer of the model performance.

Target 4, together with 1, pins down $\mu\theta^\eta$, resulting in $u_0 = 10.7\%$, $u_1 = 4.78\%$, and $\rho = 1.2$.[10] Then targets 2 and 3 pin down k and β, resulting in $k = 7.1$ and $\beta = 0.024$. The above targets also imply the relation between μ and η, or $\mu = 0.45/0.63^\eta$, not determining

[10] To see this, note that adding up (9) and (10) gives $u\mu\theta^\eta + n_1u_1\rho(1-u_1) = (1-u)\delta$. (The (LHS) is u times the job-finding rate in our model.) Target 4 implies that the first and second terms of the (LHS) are approximately equal, which pins down $\mu\theta^\eta$.

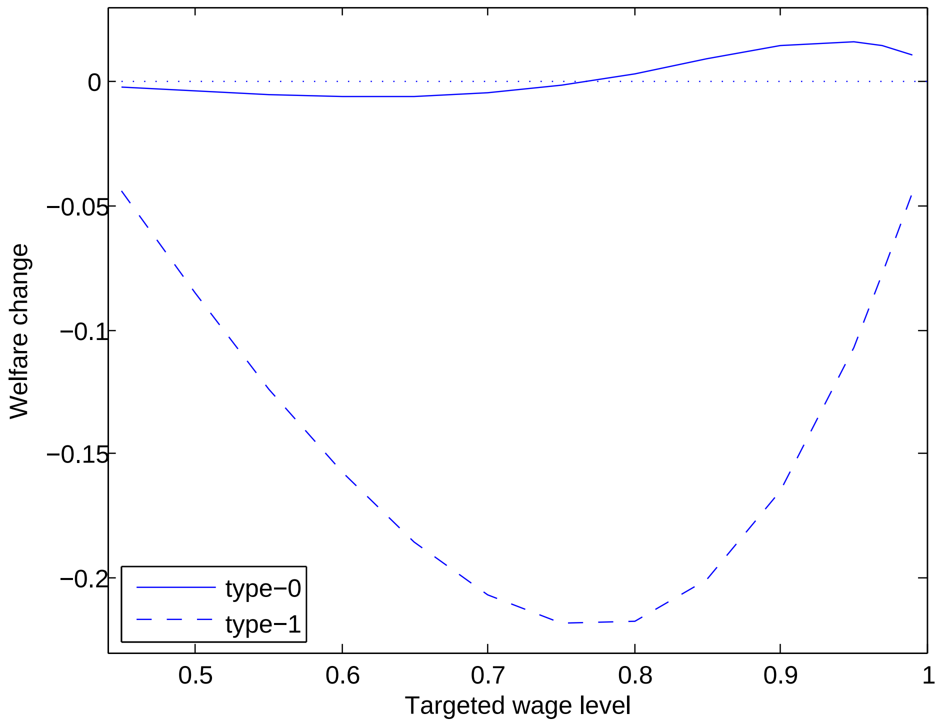

Figure 1: Welfare changes for various wage targets
Other calibration targets are maintained. η is set to be 0.3.

the two parameters separately. These two parameter values are needed to compute the post-policy equilibrium, so we try two extreme values for η, $\eta = 0.1$ and $\eta = 0.4$. In both cases, $u_m > \beta$ results, so we are in case (ii) of Proposition 3 provided that $\rho = 1.2$ is small enough.

Table 2 contains the computation results. All the lifetime values are expressed in terms of flow-value equivalence (i.e., lifetime value times r). The pre-policy ratio of the wage of those hired through networks to the average wage of those hired through the matching function is 1.08 and it matches Korenman and Turner (1996)'s observation well. Not surprisingly, the policy has large negative effects on type-1 workers. The total resource of the economy also falls with such a policy, because the unemployment rate gradually increases in the future. Moreover, the policy slightly lowers wage and values of type-0 workers.

However, in the economy where workers receive higher wages, Proposition 3(i) can also be the case. In such cases, the policy will still largely drop the total resource of the economy,

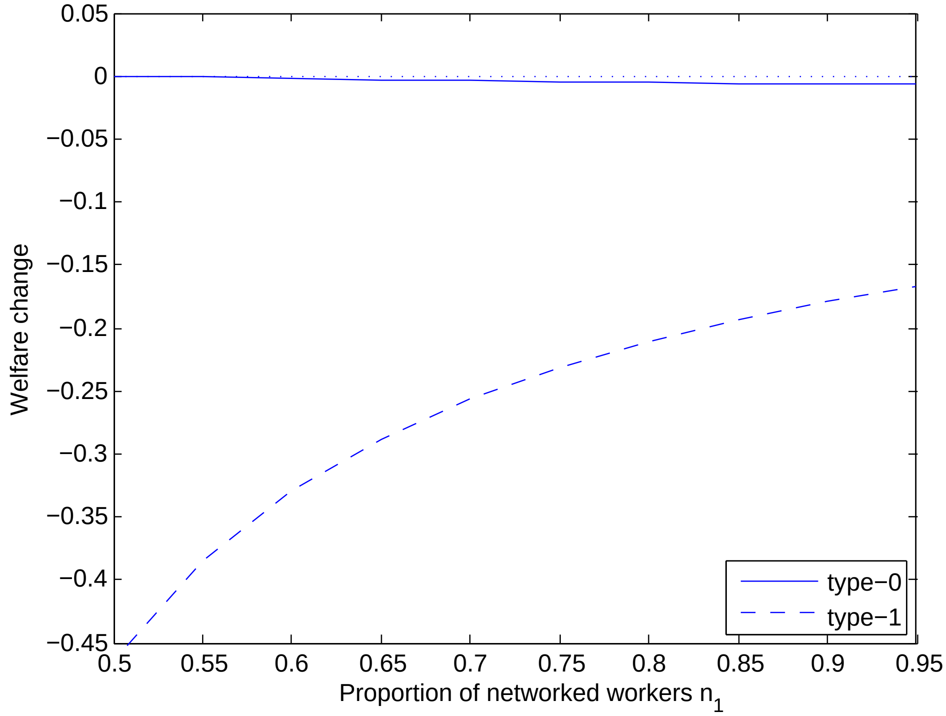

Figure 2: Welfare changes for various n_1

but make type-0 workers slightly better-off. Figure 1 shows what happens if the targeted wage levels differ from 0.666. It shows the welfare change (the post-policy lifetime value minus the pre-policy lifetime value) for each type of worker[11] As wage rises above about 0.75 (75 % of output), the change for type-0 workers turns positive, implying that the policy makes them slightly better off, yet still at the cost of the large reduction in type-1 workers' welfare.

Finally, Figure 2 shows welfare changes for each type of worker for various n_1. The figure shows that the policy effect on non-networked workers is always negative and small.

[11] The weighted average of the unemployment value and employment value are calculated. Weights are pre-policy steady-state unemployment/employment rates. Note also that as the targeted wage level varies, the calibrated values for β and k also change.

4 Stability of the steady state for $\rho > 0$

In this section, we numerically check the local stability of the Proposition-1 steady state. The dynamics of the equilibrium with positive ρ are as follows. First, the Bellman equations are[12]

$$\begin{aligned}
\dot{U}_0 &= rU_0(t) - b - \mu\theta(t)^\eta[W_0(t) - U_0(t)] && (18)\\
\dot{U}_1 &= rU_1(t) - b - (\mu\theta(t)^\eta + \rho(1 - u_1(t)))[W_1(t) - U_1(t)] && (19)\\
\dot{W}_0 &= rW_0(t) - w_0(t) - \delta[U_0(t) - W_0(t)] && (20)\\
\dot{W}_1 &= rW_1(t) - w_1(t) - \delta[U_1(t) - W_1(t)] && (21)\\
\dot{J}_0 &= rJ_0 - (y - w_0(t)) - \delta[V(t) - J_0(t)] && (22)\\
\dot{J}_1 &= rJ_1 - (y - w_1(t)) - \delta[V(t) - J_1(t)] - \rho u_1(t)[J_1(t) - V(t)] && (23)\\
\dot{V} &= rV(t) + k - \mu\theta(t)^{\eta-1}\sum_{j=0,1}\frac{n_j u_j(t)}{n_0u_0(t) + n_1u_1(t)}[J_j(t) - V(t)]. && (24)
\end{aligned}$$

The laws of motion are

$$\begin{aligned}
\dot{u}_0 &= (1 - u_0(t))\delta - u_0\mu\theta(t)^\eta && (25)\\
\dot{u}_1 &= (1 - u_1(t))\delta - u_1(\mu\theta(t)^\eta + \rho(1 - u_1(t))). && (26)
\end{aligned}$$

The Nash bargaining wage determination $[W_j(t) - U_j(t)]/\beta = [J_j(t) - V(t)]/(1 - \beta)$ and free-entry condition $V(t) = \dot{V}(t) = 0$ hold for all t. We reduce the system to that of the four variables, u_0, u_1, S_0, S_1, where $S_j \equiv W_j - U_j + J_j - V$ is the total surplus of a match with a

[12] The derivation is given at the end of Appendix.

type-j worker. First, (18)-(23) are combined to

$$\dot{S}_0 = [r + \delta + \beta\mu\theta(t)^\eta]S_0(t) - (y - b) \quad (27)$$

$$\dot{S}_1 = [r + \delta + \beta\mu\theta(t)^\eta + \rho(\beta - u_1(t))]S_1(t) - (y - b). \quad (28)$$

Also, (24), the Nash bargaining condition and the free-entry condition gives an explicit expression for $\theta(t)$: for all t,

$$\theta(t)^{1-\eta} = \frac{k}{\mu(1-\beta)}\left[\sum_{j=0,1}\frac{n_j u_j(t)}{n_0 u_0(t) + n_1 u_1(t)}S_j(t)\right]^{-1}. \quad (29)$$

In summary, (25)-(28) together with (29) gives the dynamical system in the economy with networks. Because u_0 and u_1 are state variables whose initial values are exogenously given while S_0 and S_1 are not such variables, the system is locally stable if the two or more eigenvalues of the 4×4 Jacobian (evaluated at the steady state) are negative. If exactly two are negative and the other two are positive, then the dimension of the stable manifold is two, uniquely determining the initial values for S_0 and S_1 (i.e., the system/path is "determinate"). Although one can obtain the 4×4 Jacobian of the above system analytically, getting its eigenvalues analytically is extremely complex. So we compute the eigenvalues for the model calibrated in the last section. The resulting eigenvalues are $-2.07, -0.907, 0.139$ and 0.120. Thus the fixed point is locally stable and determinate.

5 Model in which firms employ multiple workers

In Section 2, we assumed that each firm employs at most one worker. In this section, we present a model in which firms can employ multiple workers and show that such a model is reduced to one-firm-one-worker model if both production and cost of hiring satisfy constant returns to scale.

The one-firm-one-worker assumption in Section 2 led to the apparently odd referral mechanism that a network friend of a worker currently employed by firm A is referred not to firm A but to some other firm, say firm B. We also assumed that in that case firm B makes payment to firm A to the extent that firm B is indifferent between accepting and not accepting the referred worker (cf. firm A's take-it-or-leave-it offer). One may find these assumptions unnatural. In reality, a firm can employ multiple workers. Therefore, any person referred by a current employee is employed by the same firm, not by a different firm, and hence no firm-to-firm payment results. We claim below that these two environments are equivalent under an assumption that both production and cost of hiring are constant-returns-to-scale; a firm with N employees produces N times as much as a firm with a single employee, and cost to post N vacancies is N times as much as cost to post one vacancy. In other words, our model presented in Section 2 is just a fiction that simplifies the model in which firms employ more than one worker.

To see this, let $J^{\ell,m}$ be the value of a firm with ℓ non-networked workers and m networked workers that is not posting another vacancy. Let $\tilde{J}^{\ell,m}$ be the value of a firm with ℓ non-networked workers and m networked workers that is currently posting another vacancy. The free-entry condition is hence $J^{\ell,m} = \tilde{J}^{\ell,m}$ for all ℓ and m. We also make normalization $J^{0,0} = \tilde{J}^{0,0} = 0$. Bellman equations are $\forall \ell, m \geq 1$,

$$\begin{aligned} rJ^{\ell,m} &= \ell(y - w_0) + m(y - w_1) + m\rho u_1(J^{\ell,m+1} - J^{\ell,m}) \\ &+ \ell\delta(J^{\ell-1,m} - J^{\ell,m}) + m\delta(J^{\ell,m-1} - J^{\ell,m}) \end{aligned} \tag{30}$$

and

$$\begin{aligned} r\tilde{J}^{\ell,m} &= \ell(y-w_0)+m(y-w_1)-k+m\rho u_1(\tilde{J}^{\ell,m+1}-\tilde{J}^{\ell,m}) \\ &+ \ell\delta(\tilde{J}^{\ell-1,m}-\tilde{J}^{\ell,m})+m\delta(\tilde{J}^{\ell,m-1}-\tilde{J}^{\ell,m}) \\ &+ \mu\theta^{\eta-1}\frac{u_0}{u}(\tilde{J}^{\ell+1,m}-\tilde{J}^{\ell,m})+\mu\theta^{\eta-1}\frac{u_1}{u}(\tilde{J}^{\ell,m+1}-\tilde{J}^{\ell,m}), \end{aligned} \tag{31}$$

with initial conditions

$$rJ^{1,0} = y-w_0+\delta(J^{0,0}-J^{1,0}) \tag{32}$$

$$rJ^{0,1} = y-w_1+\delta(J^{0,0}-J^{0,1})+\rho u_1(J^{0,2}-J^{0,1}) \tag{33}$$

$$r\tilde{J}^{1,0} = y-w_0-k+\delta(\tilde{J}^{0,0}-\tilde{J}^{1,0})+\mu\theta^{\eta-1}\frac{u_0}{u}(\tilde{J}^{2,0}-\tilde{J}^{1,0})+\mu\theta^{\eta-1}\frac{u_1}{u}(\tilde{J}^{1,1}-\tilde{J}^{1,0}) \tag{34}$$

and

$$r\tilde{J}^{0,1} = y-w_1-k+\delta(\tilde{J}^{0,0}-\tilde{J}^{0,1})+\rho u_1(J^{0,2}-J^{0,1})+\mu\theta^{\eta-1}\frac{u_0}{u}(\tilde{J}^{1,1}-\tilde{J}^{0,1})+\mu\theta^{\eta-1}\frac{u_1}{u}(\tilde{J}^{0,2}-\tilde{J}^{0,1}). \tag{35}$$

It is easily shown that

$$J^{\ell,m} = \tilde{J}^{\ell,m} = \ell J_0 + mJ_1, \tag{36}$$

where J_0 and J_1 are from Section 2. Other equations remain the same as in the one-firm-one-worker model. So it follows that unemployment rates, wages and vacancy rate remain the same.[13] Thus our one-firm-one-worker model with the incumbent firm's take-it-or-leave-it offer when a referral arises is equivalent to one-firm-multiple-workers model with the

[13]Equation (36) also implies that firms do not get gain from mergers or dissolutions. That is, the size distribution of firms does not matter.

constant-returns-to-scale production and cost of hiring.

6 Conclusion

We consider a version of Mortensen-Pissarides model with two types of workers, workers with networks and workers without networks. The only difference between the two types is that networked workers have chance to get a job through networks in addition to through the standard matching function. A steady state equilibrium exists and under some conditions it is unique. In that steady state, networked workers have a lower unemployment rate, a higher wage and higher lifetime values than non-networked workers. Banning the use of networks in hiring reduces the economy to the Mortensen-Pissarides environment and the economy starts to converge to the MP steady state. First, it is analytically shown that such a policy can have positive or negative effects on non-networked workers, depending on the parameters. In our calibration, such a policy overall discourages firms' entry and hence is bad even for non-networked workers. Moreover, the welfare of networked workers and the total surplus of the economy both drop substantially with such a policy.

In our model, firms can use both hiring methods and do not choose whether to search for workers by means of a formal process or by means of worker networks. If firms make such decisions, one has to allow for two kinds of vacant statuses, one for firms searching via the matching function and the other for firms waiting for network matching. Such a model may have different implications and is left to be studied.

7 Appendices

For proofs, we introduce the following notations: for each $j \in \{0,1\}$,

$$S_j \equiv W_j - U_j + J_j - V \tag{37}$$

$$\alpha_{F_j} \equiv \mu\theta^{\eta-1}\frac{n_j u_j}{n_0 u_0 + n_1 u_1}. \tag{38}$$

To prove Proposition 1, we provide two lemmas.

Lemma 2.

(i) For given v, there is a unique pair (u_0^*, u_1^*) that satisfies (9)-(10); and

(ii) Such (u_0^*, u_1^*) are strictly decreasing in v.

Proof. (Lemma 2)

(i) First we prove it for the case $\delta \geq \rho$. Define the functions

$$T_0(u_0, u_1, v) \equiv u_0\mu\left(\frac{v}{n_0 u_0 + n_1 u_1}\right)^{\eta} - (1-u_0)\delta \tag{39}$$

$$T_1(u_0, u_1, v; \rho) \equiv u_1\mu\left(\frac{v}{n_0 u_0 + n_1 u_1}\right)^{\eta} + u_1(1-u_1)\rho - (1-u_1)\delta, \tag{40}$$

so that the steady-state unemployment rates given v, denoted $u_0^*(v), u_1^*(v)$, are given as the solution to $T_0(u_0^*, u_1^*, v) = 0$ and $T_1(u_0^*, u_1^*, v) = 0$.

First, we have

$$T_1(u_0, 0, v; \rho) = -\delta < 0 \tag{41}$$

$$T_1(u_0, 1, v; \rho) = \mu \left(\frac{v}{n_0 u_0 + n_1 1} \right)^\eta > 0 \tag{42}$$

$$T_{11} \equiv \frac{\partial T_1}{\partial u_1} = \mu \left(\frac{v}{n_0 u_0 + n_1 u_1} \right)^\eta \left(1 - \frac{\eta n_1 u_1}{n_0 u_0 + n_1 u_1} \right) + (1 - u_1)\rho + \delta - u_1 \rho > 0 \tag{43}$$

$$T_{10} \equiv \frac{\partial T_1}{\partial u_0} = \mu \left(\frac{v}{n_0 u_0 + n_1 u_1} \right)^\eta \frac{-\eta n_0 u_1}{n_0 u_0 + n_1 u_1} < 0 \tag{44}$$

$$T_{1v} \equiv \frac{\partial T_1}{\partial v} = \eta\mu \left(\frac{v}{n_0 u_0 + n_1 u_1} \right)^{\eta - 1} \left(\frac{u_1}{n_0 u_0 + n_1 u_1} \right) > 0. \tag{45}$$

The first three equations imply that for any $v > 0$ and $u_0 \in [0, 1]$, there is a unique $u_1 \in (0, 1)$ that satisfies $T_1 = 0$, denoted by $u_1^{T_1}(u_0; v)$.[14] Then (44) implies that $\partial u_1^{T_1}(u_0; v)/\partial u_0 = -T_{10}/T_{11} > 0$, so that $u_1^{T_1}(u_0; v)$ is increasing in u_0. Moreover, it can be shown that $u_1^{T_1}(u_0; v)$ is bounded away from 0 and 1. That is, we have

$$u_1^{T_1}(u_0\ ; v) \text{ is increasing in } u_0, \text{ and } u_1^{T_1}(0; v) > 0, \;\; u_1^{T_1}(1; v) < 1. \tag{46}$$

Similarly, we have

$$T_0(0, u_1, v) = -\delta < 0 \tag{47}$$

$$T_0(1, u_1, v) = \mu \left(\frac{v}{n_0 1 + n_1 u_1} \right)^\eta > 0$$

$$T_{00} \equiv \frac{\partial T_0}{\partial u_0} = \mu \left(\frac{v}{n_0 u_0 + n_1 u_1} \right)^\eta \left(1 - \frac{\eta n_0 u_0}{n_0 u_0 + n_1 u_1} \right) + \delta > 0 \tag{48}$$

$$T_{01} \equiv \frac{\partial T_0}{\partial u_1} = \mu \left(\frac{v}{n_0 u_0 + n_1 u_1} \right)^\eta \frac{-\eta n_1 u_0}{n_0 u_0 + n_1 u_1} < 0 \tag{49}$$

$$T_{0v} \equiv \frac{\partial T_0}{\partial v} = \eta\mu \left(\frac{v}{n_0 u_0 + n_1 u_1} \right)^{\eta - 1} \left(\frac{u_0}{n_0 u_0 + n_1 u_1} \right) > 0. \tag{50}$$

[14] When $u_0 = 0$, (40) becomes $T_1(0, u_1, v; \rho) = \mu(v/n_1)^\eta u_1^{1-\eta} + (1 - u_1)(u_1\rho - \delta)$. So (41) still holds.

Again, the first three equations imply that for any $v > 0$ and $u_1 \in [0, 1]$, there is a unique $u_0 \in (0, 1)$ that satisfies $T_0 = 0$, denoted by $u_0^{T_0}(u_1; v)$.[15] Then a similar argument leads to

$$u_0^{T_0}(u_1\ ; v) \text{ is increasing in } u_1, \text{ and } u_0^{T_0}(0; v) > 0, \;\; u_0^{T_0}(1; v) < 1. \tag{51}$$

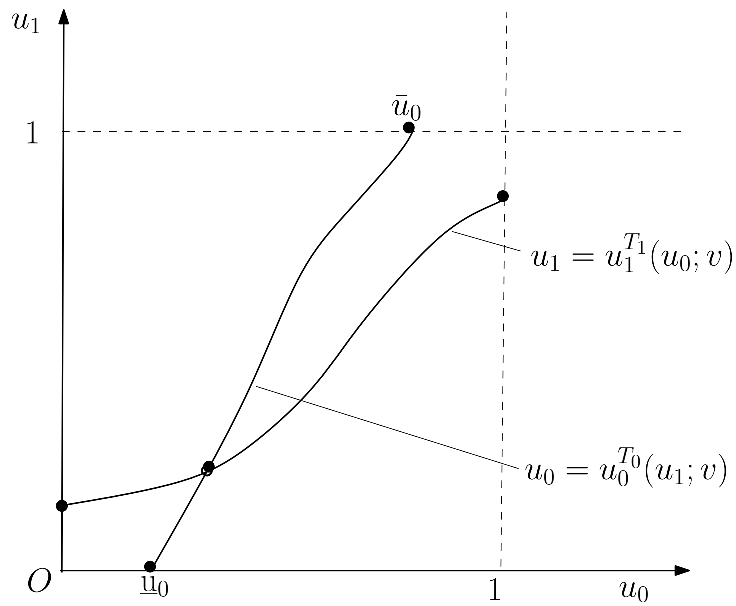

Figure 3: Graphical image of the existence proof

So let $\underline{u}_0 \equiv u_0^{T_0}(0; v)$ and $\bar{u}_0 \equiv u_0^{T_0}(1; v)$, as seen in Figure 3. Then consider the inverse function of $u_0 = u_0^{T_0}(u_1\ ; v)$ (the inverse in terms of u_1), and denote it as $u_1 = u_1^{T_0}(u_0; v)$, which is defined on $[\underline{u}_0, \bar{u}_0]$. The two functions $u_1^{T_0}(u_0; v)$ and $u_1^{T_1}(u_0; v)$ should intersect at least once because

$$u_1^{T_1}(\underline{u}_0; v) > 0 = u_1^{T_0}(\underline{u}_0; v) \tag{52}$$

$$u_1^{T_1}(\bar{u}_0; v) < 1 = u_1^{T_0}(\bar{u}_0; v) \tag{53}$$

[15] When $u_1 = 0$, (39) becomes $T_0(u_0, 0, v) = \mu(v/n_0)^\eta u_0^{1-\eta} - (1 - u_0)\delta$. So (47) still holds.

Now we are ready to show the uniqueness of such an intersection, or

$$\frac{\partial}{\partial u_0}\left\{u_1^{T_0}(u_0;v)-u_1^{T_1}(u_0;v)\right\}>0. \tag{54}$$

Note that

$$\begin{aligned}
&\frac{\partial}{\partial u_0}\left\{u_1^{T_0}(u_0;v)-u_1^{T_1}(u_0;v)\right\} \qquad (55)\\
=\;&\frac{\partial u_1^{T_0}}{\partial u_0}-\frac{\partial u_1^{T_1}}{\partial u_0}\\
=\;&\frac{T_{00}}{(-T_{01})}-\frac{(-T_{10})}{T_{11}}\\
=\;&\frac{T_{00}T_{11}-T_{10}T_{01}}{(-T_{01})\,T_{11}},
\end{aligned}$$

and the numerator is proved to be positive because by (43)-(44), (48)-(49), and the assumption $\delta>\rho$,

$$\begin{aligned}
&T_{11}T_{00}-T_{10}T_{01}\\
=\;&\left\{\mu\left(\frac{v}{n_0u_0+n_1u_1}\right)^{\eta}\left(1-\frac{\eta n_1u_1}{n_0u_0+n_1u_1}\right)+(1-u_1)\rho+\delta-u_1\rho\right\}\\
&\times\left\{\mu\left(\frac{v}{n_0u_0+n_1u_1}\right)^{\eta}\left(1-\frac{\eta n_0u_0}{n_0u_0+n_1u_1}\right)+\delta\right\}\\
&-\mu\left(\frac{v}{n_0u_0+n_1u_1}\right)^{\eta}\frac{-\eta n_0u_1}{n_0u_0+n_1u_1}\mu\left(\frac{v}{n_0u_0+n_1u_1}\right)^{\eta}\frac{-\eta n_1u_0}{n_0u_0+n_1u_1} \qquad (56)\\
=\;&(1-\eta)\left\{\mu\left(\frac{v}{n_0u_0+n_1u_1}\right)^{\eta}\right\}^2\\
&+\mu\left(\frac{v}{n_0u_0+n_1u_1}\right)^{\eta}\left(1-\frac{\eta n_1u_1}{n_0u_0+n_1u_1}\right)\delta\\
&+\mu\left(\frac{v}{n_0u_0+n_1u_1}\right)^{\eta}\left(1-\frac{\eta n_0u_0}{n_0u_0+n_1u_1}\right)((1-u_1)\rho+\delta-u_1\rho)\\
&+\delta((1-u_1)\rho+\delta-u_1\rho) \qquad (57)\\
>\;&0, \qquad (58)
\end{aligned}$$

which concludes that (55) is positive. Therefore, the two functions $u_1^{T_0}(u_0;v)$ and $u_1^{T_1}(u_0;v)$ intersect only once, which proves (i).

Next we prove the lemma for the case $\rho > \delta$. In this case, we can restrict the domain of u_1 to $[0, \delta/\rho] \subset [0,1]$ for the following reason. Consider a hypothetical situation that there is no employment through the matching function. In such a case, the dynamics of u_1 are given by $\dot{u}_1 = (1-u_1)\delta - u_1\rho(1-u_1)$, so its fixed point is $u_1 = \delta/\rho$. The steady state u_1 should be bounded from above by this level.

If we restrict attention to $u_1 \in [0, \delta/\rho]$, (43) still holds even if $\rho > \delta$. Also the last part of (46) is modified to $u_1^{T_1}(1;v) < \delta/\rho$. Additionally, the definition of $\bar{u}_0$ is modified so that $\bar{u}_0 \equiv u_0^{T_0}(\delta/\rho;v)$ and (53) is modified to $u_1^{T_1}(\bar{u}_0;v) < \delta/\rho = u_1^{T_0}(\bar{u}_0;v)$. All the other arguments remain unchanged.

(ii) Note that by the implicit function theorem,

$$
\begin{aligned}
\begin{bmatrix} \frac{\partial u_0^*}{\partial v} \\ \frac{\partial u_1^*}{\partial v} \end{bmatrix} &= -\begin{bmatrix} T_{00} & T_{01} \\ T_{10} & T_{11} \end{bmatrix}^{-1} \begin{bmatrix} T_{0v} \\ T_{1v} \end{bmatrix} \\
&= \frac{-1}{T_{11}T_{00} - T_{10}T_{01}} \begin{bmatrix} T_{11} & -T_{01} \\ -T_{10} & T_{00} \end{bmatrix} \begin{bmatrix} T_{0v} \\ T_{1v} \end{bmatrix} \qquad (59) \\
&< 0, \qquad (60)
\end{aligned}
$$

where the last inequality follows from (43)-(45), (48)-(50), and (58).

□

Lemma 3.

Given v, there exist unique S_0 and S_1 that satisfy (1)-(6) and (8)-(11), denoted $S_0^*(v)$ and $S_1^*(v)$. Both are strictly decreasing in v, satisfying $\lim_{v\to\infty} S_j^*(v) = 0$ *and* $\lim_{v\to 0} S_j^*(v) < \infty$.

Proof. (Lemma 3)

By (1), (3), (5), (8) and (11), we have

$$S_0 \quad = \quad \frac{y-b}{r+\delta+\mu\theta^\eta\beta}. \tag{61}$$

By (2), (4), (6), (8) and (11), we have

$$S_1 \quad = \quad \frac{y-b}{r+\delta+\mu\theta^\eta\beta+\rho(\beta-u_1)}. \tag{62}$$

By Lemma 2, u_0, u_1 and thus $\mu\theta^\eta$ are uniquely determined, given v. So S_0 and S_1 are unique, given v. By Lemma 2, $\mu\theta^\eta$ is strictly increasing in v, so the denominators of S_0 and S_1 in (61)(62) are both strictly increasing in v. Therefore, both S_0 and S_1 are strictly decreasing in v. Moreover, both the denominators go to infinity as $v \to \infty$ and go to some positive constants as $v \to 0$. So $\lim_{v\to\infty} S_j^*(v) = 0$ and $\lim_{v\to 0} S_j^*(v) < \infty$. □

Proof. (Proposition 1)

By Lemmas 2 and 3, given v, (1)-(6) and (8)-(11) pin down $u_0^*(v)$, $u_1^*(v)$, $S_0^*(v)$ and $S_1^*(v)$, all of which are strictly decreasing in v. The remaining equation, which determines v, is (7):

$$k = \alpha_{F_0}^*(v)(1-\beta)S_0^*(v) + \alpha_{F_1}^*(v)(1-\beta)S_1^*(v), \tag{63}$$

where for $i = 0, 1$,

$$\alpha_{F_i}^*(v) \equiv \mu\left(\frac{n_0u_0^*(v)+n_1u_1^*(v)}{v}\right)^{1-\eta}\frac{n_iu_i^*(v)}{n_0u_0^*(v)+n_1u_1^*(v)}. \tag{64}$$

When $v \to 0$, $\alpha_{F_i}^*(v) \to \infty$. So the (RHS) of (63) goes to infinity.

When $v \to \infty$,

$$\alpha_{F_i}^*(v) = \mu\frac{1}{v^{1-\eta}}\left(\frac{n_iu_i^*(v)}{n_0u_0^*(v)+n_1u_1^*(v)}\right)^\eta (n_iu_i^*(v))^{1-\eta} \to 0,$$

because the expression in the bracket is bounded above by one. So the (RHS) of (63) goes to zero. Therefore, by the intermediate value theorem, there exists a v that satisfies (63). Once v is determined, all the other equilibrium variables are uniquely pinned down. □

Proof. (Proposition 2)
First, subtracting (10) from (9) gives

$$(\mu\theta^{\eta} + \delta)(u_0 - u_1) = u_1(1 - u_1)\rho, \tag{65}$$

which implies that $u_0 > u_1$. For the rest of the proof, we consider the two cases separately: (i) $S_1 \geq S_0$ and (ii) $S_0 > S_1$. Note that combining (1) and (2) gives

$$r(U_1 - U_0) = \mu\theta^{\eta}\{(W_1 - U_1) - (W_0 - U_0)\} + (1 - u_1)\rho(W_1 - U_1); \tag{66}$$

combining (3) and (4) gives

$$w_1 - w_0 = r(W_1 - W_0) + \delta\{(W_1 - U_1) - (W_0 - U_0)\}; \tag{67}$$

combining (5) and (6) gives

$$(r + \delta)(J_1 - J_0) = w_0 - w_1 + \rho u_1(J_1 - V); \tag{68}$$

and combining (66) and (67) gives

$$w_1 - w_0 = (r + \delta + \mu\theta^{\eta})\{(W_1 - U_1) - (W_0 - U_0)\} + (1 - u_1)\rho(W_1 - U_1). \tag{69}$$

(i) Suppose $S_1 \geq S_0$ in the equilibrium. Then by the Nash bargaining condition (8), we have $W_1 - U_1 \geq W_0 - U_0$ and $J_1 \geq J_0$. Then (66) implies $U_1 > U_0$. This together with $W_1 - U_1 \geq W_0 - U_0$ implies $W_1 > W_0$. Also, (69) implies $w_1 > w_0$.

(ii) Now suppose that $S_0 > S_1$, so that the Nash bargaining condition implies $W_0 - U_0 > W_1 - U_1$ and $J_0 > J_1$. Then by (68), $w_1 > w_0$. Hence by (67), $W_1 > W_0$. This together with $W_0 - U_0 > W_1 - U_1$ implies $U_1 > U_0$.

The Proposition is proved up to this point. Lastly, we note that $S_0 > S_1$ (hence $J_0 > J_1$ and $W_0 - U_0 > W_1 - U_1$) is not necessarily the case. Subtracting (61) from (62) gives

$$(S_1 - S_0)(r + \delta + \mu\theta^\eta\beta) = S_1\rho(u_1 - \beta). \tag{70}$$

So if $u_1 < (>)\beta$, then $S_1 < (>)S_0$. □

Proof. (Lemma 1) The steady state equilibrium of the standard Mortensen-Pissarides model is given as the solution to the following system of equations:

$$\begin{aligned}
rU_m &= b + \mu\theta_m^\eta(W_m - U_m) &(71)\\
rW_m &= w_m + \delta(U_m - W_m) &(72)\\
rJ_m &= y - w_m + \delta(V_m - J_m) &(73)\\
rV_m &= -k + \mu\theta_m^{\eta-1}(J_m - V_m) &(74)\\
W_m - U_m &= \beta S_m &(75)\\
V_m &= 0 &(76)\\
u_m\mu\theta_m^\eta &= (1 - u_m)\delta, &(77)
\end{aligned}$$

where

$$S_m \equiv W_m - U_m + J_m - V_m \tag{78}$$

$$\theta_m \equiv v_m/u_m. \tag{79}$$

Again, (71)-(73) and (75), (76) are reduced to

$$S_m = \frac{y-b}{r+\delta+\beta\mu\theta_m^{\eta}}, \tag{80}$$

while (74) and (75) imply

$$S_m = \frac{k\theta_m^{1-\eta}}{(1-\beta)\mu}. \tag{81}$$

The last two equations pin down (S_m, θ_m). In fact, S_m is given as a unique positive solution to $f_S(S_m) = 0$, where

$$f_S(S_m) \equiv \beta\mu \left(\frac{\mu(1-\beta)}{k}\right)^{\frac{\eta}{1-\eta}} S_m^{\frac{1}{1-\eta}} + (r+\delta)S_m - (y-b). \tag{82}$$

Once S_m is determined, so is the v_m-u_m ratio. Meanwhile the steady state condition (77) is transformed to

$$v_m = \left[\frac{\delta}{\mu u_m^{1-\eta}} - \frac{\delta}{\mu} u_m^{\eta}\right]^{\frac{1}{\eta}},$$

which means v_m is decreasing in u_m. Combining it with (79) uniquely pins down v_m and u_m.

(Comparative Statics of the MP model.) Eliminating S_m from (80) and (81) gives

$$\frac{y-b}{k}(1-\beta) = \frac{r+\delta}{\mu\theta^{\eta-1}} + \beta\theta. \tag{83}$$

This implies that increasing β, b, or k leads to lower θ_m, resulting in higher u_m. Eliminating θ_m by using (77), one can see that $\lim_{\beta\to 0} u_m > 0$. So for sufficiently low β, $u_m > \beta$ is the case. Also, $\lim_{k\to\infty} u_m = 1$ is easy to check. □

Proof. (Proposition 3)

The equilibrium triple, (u_0, u_1, v), is pinned down by three equation. The first two are $T_0 = 0$ and $T_1 = 0$, where function T_0 and T_1 are defined by (39) and (40). The third equation, say $T_v = 0$, is defined by (7) combined with (61)(62):

$$\begin{aligned} T_v(u_0, u_1, v; \rho) &\equiv -k + \alpha_{F_0}(1-\beta)\frac{y-b}{r+\delta+\mu\theta^\eta\beta} \\ &+ \alpha_{F_1}(1-\beta)\frac{y-b}{r+\delta+\mu\theta^\eta\beta+\rho(\beta-u_1)}, \end{aligned} \tag{84}$$

where α_{F_j}, defined by (38), and $\mu\theta^\eta$ are functions of u_0, u_1, v and ρ. By the implicit function theorem,

$$\begin{bmatrix} T_{00} & T_{01} & T_{0v} \\ T_{10} & T_{11} & T_{1v} \\ T_{v0} & T_{v1} & T_{vv} \end{bmatrix} \begin{bmatrix} u_0'(\rho) \\ u_1'(\rho) \\ v'(\rho) \end{bmatrix} = - \begin{bmatrix} T_{0\rho} \\ T_{1\rho} \\ T_{v\rho} \end{bmatrix}.$$

Invoking that $u_0 = u_1 = u_m$ when $\rho = 0$, one can show that

$$u_1'(\rho)|_{\rho=0} = -\frac{u_m\left(\kappa_1 + \mu\left(\frac{v_m}{u_m}\right)^\eta (n_1\eta(1-\beta)u_m + \beta(1-n_1\eta)(1-u_m))\right)}{\kappa_2\kappa_3},$$

where

$$\begin{aligned}
\kappa_1 &\equiv (r+\delta)(1-\eta)(1-u_m) > 0 \\
\kappa_2 &\equiv \delta + \mu\left(\frac{v_m}{u_m}\right)^\eta > 0 \\
\kappa_3 &\equiv (r+\delta)(1-\eta) + \beta\mu\left(\frac{v_m}{u_m}\right)^\eta > 0 \\
\kappa_4 &\equiv r+\delta+\beta\mu\left(\frac{v_m}{u_m}\right)^\eta > 0,
\end{aligned}$$

so u_1 is decreasing in ρ near $\rho = 0$.

(ii) Calculations similar to the above lead to

$$\begin{aligned}
u_0'(\rho)|_{\rho=0} &= \frac{n_1\eta\mu\left(\frac{v_m}{u_m}\right)^\eta u_m(\beta-u_m)}{\kappa_2\kappa_3} \\
v'(\rho)|_{\rho=0} &= \frac{n_1 v\{-\kappa_1-\kappa_2(\beta-u_m)-\mu\left(\frac{v_m}{u_m}\right)^\eta(\eta(1-\beta)u_m+\beta(1-\eta)(1-u_m))\}}{\kappa_2\kappa_3}
\end{aligned}$$

Using (61) and (62),

$$\begin{aligned}
S_0'(\rho)|_{\rho=0} &= \frac{n_1\eta\beta S_m\mu\left(\frac{v_m}{u_m}\right)^\eta(\beta-u_m)}{\kappa_3\kappa_4} \\
S_1'(\rho)|_{\rho=0} &= \frac{(u_m-\beta)S_m((r+\delta)(1-\eta)+\beta\mu\left(\frac{v_m}{u_m}\right)^\eta(1-n_1\eta))}{\kappa_3\kappa_4}.
\end{aligned}$$

If $u_m < \beta$, we have $u_0'(\rho)|_{\rho=0} > 0$, $v'(\rho)|_{\rho=0} < 0$, $S_0'(\rho)|_{\rho=0} > 0$, and $S_1'(\rho)|_{\rho=0} < 0$. Then (8) implies that $W_0 - U_0$ and J_0 are increasing and that $W_1 - U_1$ and J_1 are decreasing in ρ. By (5), $w_0 = y - (r+\delta)J_0$, so w_0 is decreasing. Then by (3), W_0 is decreasing as well. $W_0 - U_0$ increasing and W_0 decreasing imply U_0 decreasing. Also, u_0 increasing and (9) imply that $\mu\theta^\eta$ is decreasing. Hence θ is decreasing.

(iii) If $\beta < u_m$, we have $u_0'(\rho)|_{\rho=0} < 0$, $S_0'(\rho)|_{\rho=0} < 0$, and $S_1'(\rho)|_{\rho=0} > 0$, while the sign of $v'(\rho)|_{\rho=0}$ is indeterminate. The results are then opposite of those of (ii) except for v. Moreover, w_0, W_0 and U_0 being increasing and Proposition 2 imply that w_1, W_1 and U_1 are also increasing near $\rho = 0$. Also, because both u_0 and u_1 are decreasing, u is decreasing. □

Proof. (Proposition 4)

(i)(ii) We want to show that (63) in the proof of Lemma 1 has unique solution v. For that, it suffices to show that (64) is strictly decreasing. Differentiating (64) with respect to v and using (59), (43)-(45) and (48)-(50), we have

$$\frac{\partial \alpha^*_{F_0}(v)}{\partial v} = -\frac{n_0 u_0 \mu\theta^\eta \left\{ \begin{array}{l} n_0 u_0 (1-\eta)(\delta + \mu\theta^\eta)(\delta + \mu\theta^\eta + q) \\ + n_1 u_1 \left[\delta^2 (1-\eta) + \delta(1-\eta)(2\mu\theta^\eta + q) + \mu\theta^\eta(\mu\theta^\eta(1-\eta) + q)\right] \end{array} \right\}}{v^2 \{n_0 u_0 (\delta + \mu\theta^\eta(1-\eta))(\delta + \mu\theta^\eta + q) + n_1 u_1 (\delta + \mu\theta^\eta)(\delta + \mu\theta^\eta(1-\eta) + q)\}},$$

and

$$\frac{\partial \alpha^*_{F_1}(v)}{\partial v} = -\frac{n_1 u_1 \mu\theta^\eta \left\{ \begin{array}{l} n_1 u_1 (1-\eta)(\delta + \mu\theta^\eta)(\delta + \mu\theta^\eta + q) \\ + n_0 u_0 \left[\delta^2 (1-\eta) + \delta(1-\eta)(2\mu\theta^\eta + q) + \mu\theta^\eta(\mu\theta^\eta(1-\eta) + q(1-2\eta))\right] \end{array} \right\}}{v^2 \{n_0 u_0 (\delta + \mu\theta^\eta(1-\eta))(\delta + \mu\theta^\eta + q) + n_1 u_1 (\delta + \mu\theta^\eta)(\delta + \mu\theta^\eta(1-\eta) + q)\}},$$

where $q \equiv (1 - 2u_1^*(v))\rho$. In Lemma 2, we showed that $u_1^*(v)$ is strictly decreasing in v, so the above two equations imply that the $\alpha^*_{F_i}(v)$'s are strictly decreasing in the region such that $q \geq 0$ or $u_1^*(v) \leq 0.5$. So, the (RHS) of (63) is also strictly decreasing in v in the region such that $u_1^*(v) \leq 0.5$, as is shown in Figure 4. Therefore, there is at most one equilibrium such that $u_1 \leq 0.5$. The (RHS) of (63) does not necessarily have humps as is depicted in Figure 4. All we know is that it goes to infinity as $v \to 0$, goes to zero as $v \to \infty$, and it is strictly decreasing for sufficiently large v. Parameter k appears only in this equation, so if k

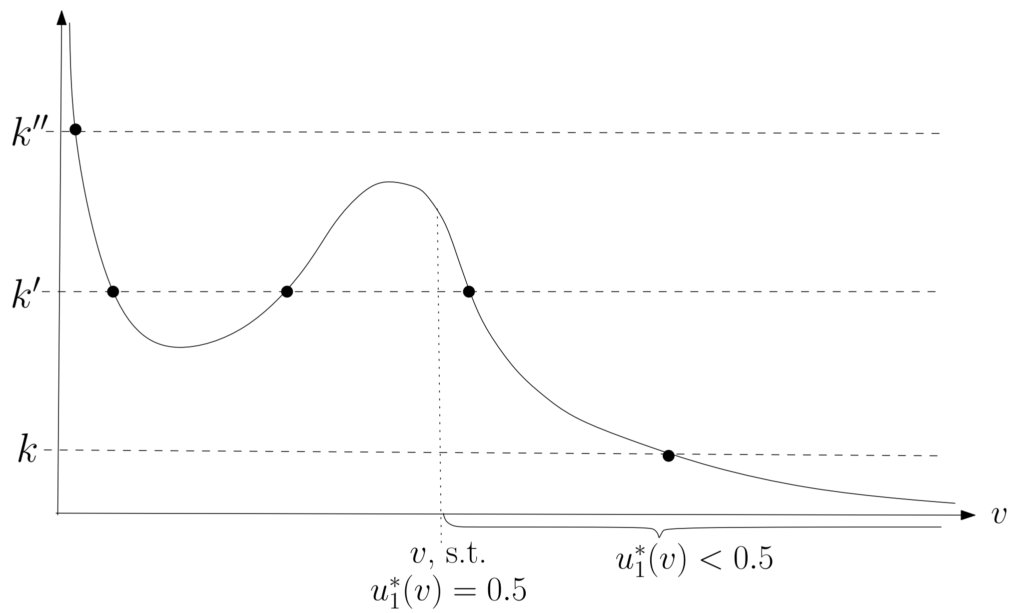

Figure 4: The (RHS) of equation (63)

is large enough, the equilibrium is unique and $u_1 > 0.5$. If k is small enough, the equilibrium is unique and u_1 can be any small number.

(iii) Proposition 3(i) implies that if $\rho \approx 0$, then $u_1 < u_m$ is the case in any equilibrium. Then part (i) implies the uniqueness of the steady state equilibrium.

□

Proof. (Proposition 5)

Defining the surplus of a match $S_m(t) = W_m(t) - U_m(t) + J_m(t) - V_m(t)$, the Nash bargaining condition and the free-entry condition together with (12)-(14) imply

$$\dot{S_m} = (r + \delta + \beta\mu\theta_m(t)^\eta)S_m(t) - (y - b). \tag{85}$$

In the meantime, (15), the Nash bargaining condition and the free-entry condition imply

$$\theta_m(t) = \left(\frac{\mu(1-\beta)}{k}\right)^{\frac{1}{1-\eta}} S_m(t)^{\frac{1}{1-\eta}}, \quad \forall t. \tag{86}$$

Substituting the last equation into (85), we have

$$\dot{S_m} = \beta\mu \left(\frac{\mu(1-\beta)}{k}\right)^{\frac{\eta}{1-\eta}} S_m(t)^{\frac{1}{1-\eta}} + (r+\delta)S_m(t) - (y-b) \equiv f_S(S_m(t)). \tag{87}$$

So the match surplus is autonomous, not depending on other variables such as the unemployment rate. Since $f_s'(S_m) > 0$, the match surplus should be constant at the steady state level for any convergent path. So, $\theta_m(t)$ is also constant. But then, (16) is an autonomous system with the (RHS) having negative slope. Therefore, $u_m(t)$ is convergent. By the Nash bargaining condition, $S_m(t)$ being time-invariant implies that $W_m(t) - U_m(t)$, $J_m(t) - V_m(t)$ and $J_m(t)$ are also time-invariant. Then (14) implies $w_m(t)$ is time-invariant. So (12) and (13) become an autonomous system of $U_m(t)$ and $W_m(t)$, respectively. Since the slopes of these systems are positive, $U_m(t)$ and $W_m(t)$ should also be at the steady-state levels from the beginning. □

Proof. (Proposition 6)
Define $Y_m(t) \equiv u_m(t)U_m(t) + (1-u_m(t))(W_m(t) + J_m(t))$. This is equal to $W_m(t) + J_m(t) - u_m(t)S_m(t)$. Therefore,

$$\begin{aligned}
\dot{Y_m} &= \dot{W_m} + \dot{J_m} - \dot{u_m}S_m - u_m\dot{S_m} \\
&= r(W_m + J_m) - y - \delta(U_m - W_m + V_m - J_m) - \dot{u_m}S_m - u_m[(r+\delta+\beta\mu\theta_m^\eta)S_m - (y-b)] \\
&= -bu_m - y(1-u_m) + r(W_m + J_m - u_mS_m) - \dot{u_m}S_m + [\delta(1-u_m) - u_m\mu\theta_m^\eta]S_m \\
&\quad +u_m(1-\beta)S_m\mu\theta_m^\eta \\
&= -bu_m - y(1-u_m) + r(W_m + J_m - u_mS_m) + u_m(1-\beta)S_m\mu\theta_m^\eta \\
&= -bu_m - y(1-u_m) + kv_m + r(W_m + J_m - u_mS_m) \\
&= -bu_m - y(1-u_m) + kv_m + rY_m,
\end{aligned}$$

where the second equality uses (13), (14) and (85), the fourth equality uses (16), and the fifth uses (86). The last expression implies

$$Y_m(t) = \int_t^{\infty} e^{-r(\tau-t)}[bu_m(\tau) + y(1-u_m(\tau)) - kv_m(\tau)]d\tau,$$

that is, the discounted sum of the economy's total resources. □

Proof. (Derivation of differential equations in section 4)

Suppose $\tau \geq 0$ is a random variable that is the time until an event occurs. Let $\Phi(\tau)$ and $\phi(\tau)$ be its distribution function and density function, respectively. That is,

$$\begin{aligned}
\Phi(t_2|t_1) &\equiv \mathbb{P}(\tau \leq t_2|\tau > t_1) \\
&= \mathbb{P}(t_1 < \tau \leq t_2|\tau \geq t_1) \\
&= \frac{\Phi(t_2) - \Phi(t_1)}{1 - \Phi(t_1)}.
\end{aligned}$$

Differentiating with respect to t_2, we have

$$\phi(t_2|t_1) \equiv \phi(t_2|\tau > t_1) = \frac{\phi(t_2)}{1 - \Phi(t_1)}.$$

Let $\lambda(\tau) \equiv \phi(\tau)/(1 - \Phi(\tau))$, the hazard rate. Then we have

$$\frac{\partial \phi(t_2|t_1)}{\partial t_1} = \lambda(t_1)\phi(t_2|t_1), \tag{88}$$

which we often use below. Another formula we often use is as follows. Let

$$D(t, T) \equiv \int_t^T f(s) e^{-r(s-t)} ds,$$

where f is a given function. Then

$$\frac{\partial}{\partial t}D(t,T) = -f(t) + rD(t,T). \tag{89}$$

□

The type-0 worker's unemployment value satisfies

$$U_0(t) = \int_t^\infty \phi(T|t)D(t,T)dT + \int_t^\infty \phi(T|t)e^{-r(T-t)}W_0(T)dT,$$

where $f(s) = b$ is a constant function and $\phi(T|t)$ is a density for arrival of employment. (So, $\lambda(t) = \mu\theta(t)^\eta$.) Differentiating it with respect to t and using (88)(89), we have

$$\dot{U}_0(t) = rU_0(t) - b - \lambda(t)[W_0(t) - U_0(t)]. \tag{90}$$

The derivation is similar for the other values.

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